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UNISEXUAL PROGENIES AND THE SEX CHROMOSOME MECHANISM IN SCIARA[1]

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The regular production of "unisexual" progenies, characteristic of many species of *Sciara*, has been considered in earlier papers. In these it has been shown (1) that the sex of the progeny as a whole is determined by the female,[2] (2) that "sex of progeny" is a distinct genetic character inherited in a definite fashion[3] and (3) that the female-producing female breeds as if heterozygous and the male-producing female as if homozygous recessive, for the gene or gene complex responsible for this character.[3] In addition it has been shown (1) that the sex of the individual fly, as distinguished from that of the progeny as a whole, appears to be dependent on an XX-XY sex chromosome mechanism, the male being XY and the sperms, therefore "sex-determining,"[4] (2) that each male apparently produces X-bearing and Y-bearing sperms in equal numbers,[4] and (3) that the "unisexual" constitution of the progenies appears to result from a process of selective fertilization which causes the eggs of female-producing females to be fertilized only by X-bearing sperms and those of male-producing females only by Y-bearing sperms.[5]

From these observations it appears that the sex of the individual fly depends on the type of sperm fertilizing the egg, but that as regards the eggs of any one female the type of sperms so functioning is in turn predetermined by the zygotic constitution of the female herself. In the present paper evidence is considered which indicates that both of these characteristics—the sex of the individual fly and the sex of the progeny as a whole—are "determined" by the sex chromosomes and that as regards this particular feature, at least, we are dealing with three different kinds of sex chromosomes.

It should be observed in this connection that the sex chromosomes considered here do not include the peculiar, large, "sex-limited" or "male-limited" chromosomes found only in males—considered in earlier papers.[6]

These latter chromosomes apparently have no direct effect on sex determination.

The species used in the experiments recorded here is *S. coprophila* Lintner, the same as that used in most of the earlier work. The data upon which the present summary is based will be considered in detail in another paper. They have been obtained from the study of two sex-linked, recessive, mutant characters, "swollen" and "narrow."

These characters are inherited in the manner indicated in figure 1. For example, when swollen males are outcrossed to unrelated wild-type females the offspring are all wild type. When the F_1 females are backcrossed to swollen males the resulting progenies include wild-type and swollen in a 1:1 ratio as expected, save for a deficiency of swollen females due to poor viability. When the females of this generation are tested the swollen ones all prove to be male-producers and give exclusively swollen sons, aside from the occasional "exceptional" daughters. Conversely,

1. *Wild-type female* × *swollen male.*
F_1. All wild-type.

2. *Backcross, F_1 female* × *swollen male.*

Male progenies.	*Female progenies.*
Wild-type and swollen	Wild-type and swollen
1:1 ratio.	1:1 ratio.

3. *Tests of wild-type and swollen sisters from 2.*

Swollen female	*Heterozygous female.*
(by wild-type or swollen male)	(by swollen male)
Only male progenies.	Only female progenies.
Swollen males.	Wild-type and swollen, 1:1 ratio.

FIGURE 1
Inheritance of "swollen" in *S. coprophila*

their wild-type sisters are all female-producers and, if crossed to swollen males, give the same results as their mothers. This process may be repeated over and over. Judging from present evidence it will continue indefinitely, the swollen females regularly being male-producers and their heterozygous sisters female-producers.

The same type of inheritance is exhibited by "narrow," i.e., narrow females regularly prove to be male producers while their heterozygous sisters prove to be female-producers. Both characters have been used on a large enough scale to establish the essential constancy of this behavior. In the case of swollen, extensive experiments have been carried on over a period of more than fifteen generations. These have included both pair matings and mass matings, all designed to reveal any exceptions to the above type of behavior. In the case of narrow the tests have been similar, but less extensive. All together five hundred or more females have been tested singly and probably more than that number in mass

cultures. In two cases females recorded as swollen gave progenies recorded as female. The discrepancy here was not noted in time to test the flies, but it is probable that both cases are either due to errors in recording or to the use of mosaic flies which were genetically female and heterozygous, but whose wings were of the male type and swollen. Such individuals are not uncommon in this material. With these two exceptions the results all conform to the scheme outlined above.[7]

Considering the above evidence in connection with that reported in earlier papers, we interpret the sex chromosome mechanism here as follows: (1) The male is XY. (2) The male-producing female is homozygous for the X found in the male, thus being XX. (3) The female-producing female has one X of this kind, together with a chromosome which differs from X at least in respect to the agent responsible for "sex of progeny." The latter is designated X′. The X′ chromosome, or the portion of it

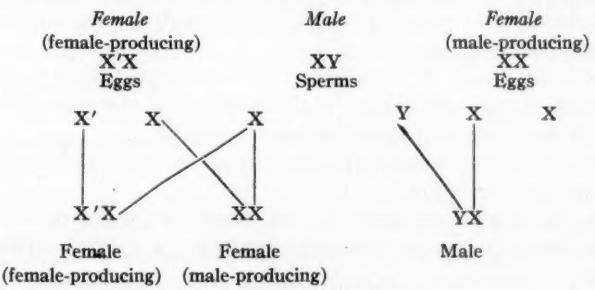

FIGURE 2

Diagram illustrating mode of sex determination in *Sciara*

considered here, ordinarily descends directly from mother to daughter, thus being sex-limited in its transmission somewhat like the Y-chromosome, except that it goes to only half the daughters. The X-chromosome, on the other hand, passes back and forth between the male and both kinds of females. These features are shown schematically in figure 2.

On this basis the differential responsible for "sex of progeny" is to be considered as a gene or gene complex in the sex chromosome—that in the X-chromosome being recessive to that in X′. Interest attaches, of course, to the nature of this differential. In the preliminary stages of the work, here reported, an especial effort has been made to ascertain whether it is a single gene (pair) or a composite involving a considerable part of the chromosome. If the former, it should exhibit crossing-over with other genes in the chromosome. As already indicated, however, no cross-overs have been detected between it and either of the two sex-linked mutant genes used. The latter both arose in the X-chromosome and we have been

unable, thus far, to get either into the X′ chromosome (or the part under consideration).

The significance of this fact is not yet clear, for the two genes prove to be very closely linked when tested together in the XX female. Here they give considerably less than one per cent crossing-over—probably not more than one half per cent. If, therefore, sex of progeny were determined by a gene lying very close to that for swollen, on the chromosome, it is possible that, with the same rate of crossing-over as in the XX female, we would have failed to detect any interchange. In this case cross-overs ought to be detectable between that gene and the gene for narrow, which has not yet been used very extensively. Experiments are now under way to test this possibility. If they fail to give cross-overs the evidence will be strong that crossing over between X and X′ does not occur as readily as between X and X.

Leaving this feature aside, we may sum up present evidence bearing on the relationship between X and X′ as follows (including data presented in the accompanying paper):

1. XX females, X′X females and X′X′ females appear to be indistinguishable somatically, except by the introduction of mutant characters.
2. XY males and X′Y males are likewise similar.
3. X′ carries the normal allelomorphs of the two sex-linked mutant genes thus far detected in X.

From this evidence we conclude that, whatever the differences may be between them, the two chromosomes must have a common fundamental structure.

[1] This investigation has been aided by a grant from the NATIONAL RESEARCH COUNCIL, Committee for Research in Problems of Sex.

[2] Moses and Metz, '28, these PROCEEDINGS, **14**, 928–930. In the present paper the term "sex of progeny" is used in preference to "sex ratio," since it is the sex of the progeny as a whole and not the exact ratio, which is being considered.

[3] Metz and Moses, '28. These PROCEEDINGS, **14**, 930–932.

[4] Metz and Ullian, '29. *Ibid*, **15**, 82–85.

[5] Metz, '29, *Amer. Nat.*, **63**, 214–228.

[6] Metz and Ullian, loc. cit. Also Metz, Moses and Hoppe. *Zeit. induk. Abstammungs- u. Vererb.*, **42**, 237–270.

[7] The apparently conflicting results of the early experiments on swollen, as recorded by Metz and Ullian, '29 (loc. cit.), will be considered in a forthcoming paper. They were due in part to confusing swollen with another character and in part to the effects of nondisjunction, which at that time was not known to be frequent in this material.

FURTHER STUDIES ON THE CHROMOSOME MECHANISM RESPONSIBLE FOR UNISEXUAL PROGENIES IN SCIARA. TESTS OF "EXCEPTIONAL" MALES[1]

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On the basis of genetic evidence presented in earlier papers[2] an interpretation of sex determination in *Sciara* has been developed which includes, among others, the following features:

(1) Speaking broadly, sex determination involves two separate elements: one determination of the sex of the individual fly; the other determination of the sex of the progeny as a whole—the progenies regularly being "unisexual" in the species under consideration.

(2) "Sex of progeny" is a distinct genetic character, exhibiting a definite type of inheritance.

(3) Both sex of progeny and sex of the individual are "determined" by the same pair of chromosomes—the sex chromosomes.

(4) Three "ordinary" types of individuals are found, each differing from the other two in respect to its sex chromosomes. Males are XY, male-producing females are XX and female-producing females are XX′.

(5) Thus the X chromosome is represented in all three types, while ordinarily the Y chromosome is limited to males and the X′ chromosome to female-producing females.

(6) The X′ chromosome differs from X in respect, at least, to the differential (gene or genes?) responsible for sex of progeny, i.e., it carries something which makes a female a female-producer instead of a male-producer. No other certain difference has been detected thus far between X′ and X, possibly because no crossing-over has been detected between them.

(7) In male progenies occasional "exceptional" females appear and similarly in female progenies "exceptional" males may occur.

(8) Since exceptional females come from male-producing (XX) mothers they should never receive an X′ chromosome and hence should all be male-producers. This conclusion has been verified.[3]

(9) Exceptional males, coming as they do from XX′ mothers, should, theoretically, be of two types, one the ordinary XY, the other a fourth type of individual, X′Y. The latter, if viable and fertile, should give distinctly different genetic results from the former, when crossed to female-producing females. One such result would be the production of a fifth type of individual—an X′X′ female.

The present paper deals with evidence showing that X′Y males may

be secured and that they exhibit the expected type of genetic behavior, including the production of X′X′ females.

By using one of the available sex-linked mutant characters, swollen or narrow, it is possible to distinguish XX′ females from their XX sisters, as shown elsewhere.[4]

In the XX′ females thus obtained the X-chromosome carries the gene for swollen or narrow. Mating such females to swollen or narrow males makes it possible to distinguish between exceptional sons of the desired X′Y type and those of the ordinary XY type. The former are wild-type in appearance, while the latter are swollen or narrow, as the case may be. The males tested in the present experiments were obtained in this manner. Two sets of experiments were made, as recorded below.

In one (referred to as line 4449) an X′Y male was mated to three virgin female-producing females heterozygous for narrow (X′Xn). One hundred thirteen F_1 offspring were obtained, all wild-type females as expected. Since these females should all receive an X′ chromosome from their fathers they should all be female-producers, instead of half being male-producers as would ordinarily be the case. They should be of two genetic types, half X′X′ and half XnX′, as shown by the following scheme:

P_1 X′Xn female by X′Y male
F_1 X′X′ and XnX′, all females

The former (X′X′) should, in turn, give only female-producing daughters, while the others should give daughters of the usual two kinds, half female-producers and half male-producers.

Seventeen of the F_1 females mentioned above were tested singly by mating to swollen males, with the expected results. The progeny counts here ranged from 44 to 130, with a total of 1558 wild-type females and 1 wild-type "exceptional" male.

According to the above scheme these progenies should be of two kinds—half composed solely of female-producing females and half composed of female-producers and male-producers in equal numbers. Females from sixteen of the progenies were tested singly. The records are too extensive to be given in detail here, but may be summarized as follows:

Taking the progenies in sequence the number of females tested from each is, *a*—15, *b*—12, *c*—17, *d*—9, *e*—15, *f*—16, *g*—8, *h*—16, *i*—3, *j*—5, *k*—6, *l*—12, *m*—17, *n*—12, *o*—12, *p*—9. Out of these sixteen series, nine (*a*, *b*, *f*, *i*, *l*, *m*, *n*, *o*, *p*) were exclusively composed of female progenies, while seven (*c*, *d*, *e*, *g*, *h*, *j*, *k*) were composed of male progenies and female progenies in approximately equal numbers. In other words, nine of the F_1 females proved to be X′X′ and seven XX′, if we count all the series just mentioned. Three of the series (*i*, *j*, *k*) contained only 3, 5 and 6 progenies, respectively. If we omit these the ratio becomes 6X′X′:7XX′.

In either case there is a close approximation to the expected equality.

Following the generation just considered there should, on our scheme, be no further peculiarities in genetic behavior due to the original X′Y male used. Females in female-progenies descended from X′X′ grandmothers should be just like those in female-progenies derived from XX′ grandmothers. In both cases half should be male-producers and half female-producers. To test this, matings were made from both sources. Forty-one females, from six progenies of X′X′ descent gave twenty-two female progenies to nineteen male progenies (ratios 3:3, 6:2, 7:3, 2:2, 3:4 1:5, respectively) and twenty-three females of XX′ descent gave fourteen female to nine male progenies (0:1, 7:6, 0:1, 7:1, respectively). Here again the results agree with expectation.

Among the male progenies in the second and third generations considered above were several in which an unusually large number of "exceptional" females appeared. At first it was suspected that the cause of these might be traceable to the original X′Y male used, but subsequent tests have not borne this out. Likewise the suspicion is not supported by evidence from tests of the other X′Y male, as noted below. Descendants of these females are still under observation, however, and their behavior will be considered in a subsequent, more detailed, account.

The second series of experiments (4676 line) began with a mating of an X′Y male to female-producing females heterozygous for swollen (X′Xs). Their descendants were tested in a manner similar to that used in the preceding experiments and with similar results, although the tests were much less extensive. Offspring in the first and second generation consisted entirely of females. Only four second-generation progenies were tested and two of these tests involved numbers too small to be significant. The other two, however, clearly represent the two expected types. One lot gave twelve female progenies and no male progenies—evidently having descended from an X′X′ female. The other gave six female and two male progenies—indicating descent from an XX′ female. In the next generation the usual type of results were obtained in tests from both lots. Out of the eleven progenies descended from the X′X′ female six were female and five male. In the other lot six were female and eight male.

In this entire series of experiments only one exceptional female and eighteen exceptional males were secured—the largest number from any one mother being two. There was no trace of the tendency toward excess production of exceptional females observed in the preceding series of experiments.

The above observations, taken in connection with the tests of exceptional females reported in an earlier paper, serve to confirm the interpretation there presented[3] of the chromosome mechanism underlying the production of unisexual progenies in *Sciara*. They also serve to throw some

light on the nature of the X′ chromosome and its relation to X. Perhaps the most significant feature in this latter respect is the fact that X′ may be substituted for X in both males and females without any conspicuous effect on the external morphology, viability or fertility of the flies, although it is evident that in nature the special combinations considered here (X′Y males and X′X′ females) are relatively uncommon. This interchangeability indicates a basic likeness in constitution between X′ and X, as noted more specifically in the accompanying paper.[2]

Although the present study, like most of those preceding it, is based on *Sciara coprophila*, it seems probable that the conclusions here apply also to the other species of the genus which give unisexual progenies.

The authors are indebted to Dr. Helen Monosmith for aid in making many of the counts in the above experiments.

[1] This investigation has been aided by a grant from the NATIONAL RESEARCH COUNCIL, Committee for Research in Problems of Sex.

[2] Summarized, for the most part, in the accompanying paper, these PROCEEDINGS, **15**, 863–866 (1929).

[3] Metz and Moses, '28. These PROCEEDINGS, **14**, 930 (1928).

CHANGES IN THE RATE OF MUTABILITY OF THE MUTABLE MINIATURE GENE OF DROSOPHILA VIRILIS

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All mutable miniature flies trace their origin to a single male found among the offspring of a pair mating in 1925. Since miniature is a sex-linked character and since Drosophila males are haploid for the X-chromosome all miniature flies trace their origin to a single gene. In the first offspring of the original miniature male the miniature gene was mutable to wild type both in the germ cells and in the somatic cells.[1] Later, however, changes occurred in the mutable miniature gene, which made it possible to isolate, in addition to the line mutable both in the germ cells and the somatic cells (alpha-line), a miniature line mutable in somatic cells only (gamma line) and another line in which miniature behaved as an almost constant gene (beta line). In this paper it is intended to consider the behavior of these three lines of the mutable miniature, especially the last two.

MINIATURE-ALPHA LINE. As already mentioned, the miniature gene of the miniature-alpha line is mutable both in the germ cells and in the somatic cells. Mutations affecting the germ cell produce wild-type individuals; and those occurring in the cells of the wings make the wings mosaics of wild-type and miniature tissue.

The data supporting the above assumptions have been summarized in previous papers[1,2] and are considered sufficient for this paper.

MINIATURE-BETA LINE. *Origin.*—Because of the changes in the miniature gene, miniature-alpha flies when inbred give wild-type, mosaic and miniature offspring. The wild-type flies originate through reversions of the miniature gene to its wild-type allelomorph in the germ cells or in the tissue from which the germ cells develop. The wild-type allelomorph of miniature, which originated through reversion of the mutable miniature gene, behaved in all tests as a constant one. The unstable miniature-alpha gene, therefore, changes frequently to a stable wild type gene.

The question arose whether the miniature-alpha gene ever changes in the opposite direction, viz., whether it ever becomes a constant miniature gene. To test that possibility an experiment was performed as follows: Five miniature-alpha males were mated each with a virgin miniature-1[2] forked-2 female and the female offspring of that mating were mated individually to miniature-gamma males homozygous for the *S*-1 gene, which gene stimulates the somatic mutability. Since miniature is a sex-linked character and males of Drosophila are haploid for the X-chromosome, all of the female offspring of a male of the first mating received miniature-alpha gene which traced its origin to one single gene in the father. The object of this first mating was to obtain a number of females carrying in a heterozygous condition genes derived from a single miniature-alpha gene since in that condition they could be readily tested for constancy. The second mating of the experiment was the test for constancy of miniature-alpha. In case no change in the mutable miniature occurred, about half of the males of the second mating, carrying miniature-alpha, would be expected to be mosaics because of the presence of the *S*-1 gene which was introduced into the cross through the fathers and which is known to stimulate the somatic mutability of the mutable miniature.[3] The other half of the males, those carrying miniature-1, would be expected to be miniature. In case miniature alpha changed in the male of the first cross of the experiment into a constant miniature gene, then all of the male offspring of the females which received such changed gene should be miniature. However, if a lethal mutation close to miniature-alpha occurred in any of the females of the second mating that mutation would eliminate the majority of mosaic males and leave miniature-1 males only. Such culture would have the appearance of a culture having miniature-alpha changed to a constant miniature. To avoid any possible mistake

due to that cause forked-2 was introduced into the cross together with miniature-1. Since forked-2 is closely linked with miniature-1 any elimination of miniature-alpha classes could be noticed by the absence of the non-forked class.

A summary of the results of that experiment is given in table 1 from which it may be seen that out of a total of 72 miniature-alpha miniature-1 forked-2 females tested, 71 gave half of the males mosaic and the other half miniature and only one female produced all miniature males. This female gave 16 miniature-1 forked-2 and 34 miniature males indicating that the absence of mosaic males was not due to the elimination of the

TABLE 1

THE RESULTS OF *mt-α/mt*-1 $F_2 \times$ *S*-1/*S*-1 CROSSES

	NUMBER OF FEMALES PRODUCING MALES		
F_1 CULTURE NUMBER	HALF MOSAICS HALF MINIATURE	ALL MINIATURE	TOTAL
7496	7	–	7
7497	26	–	26
7498	6	–	6
7504	21	–	21
7507	11	1	12
Total	71	1	72

miniature-alpha classes but was caused by a change in the mutable gene from a mutable alpha form to a constant beta form. This change occurred once among 72 genes tested.

Constancy of Miniature-Beta.—The offspring of beta flies obtained in the experiment described above were tested for constancy by mating miniature-beta females to males homozygous for the gene *S*-1. As already mentioned *S*-1 stimulates the somatic mutability of mutable miniature, and if beta were not constant all males of that mating would be expected to be mosaics. From 309 such matings a total of 31 mosaics and 9625 miniature males were obtained indicating that miniature-beta behaved as an almost constant miniature. Ten of the mosaic males were tested further by mating to miniature-1 females, eight of them gave miniature offspring only, and two were sterile. Eight out of ten of these mosaic males, therefore, were genetically beta but had a few reverted cells in the somatic tissues of the wings. Two sterile males were probably primary non-disjunctions.

During seventeen generations miniature beta flies were propagated by pair matings. During that time 83 cultures produced 11609 miniature and 5 mosaic flies indicating again a very high degree of constancy of miniature-beta.

Changes in the Miniature-Beta Gene.—After having been propagated by mass matings for fourteen months, or about seventeen generations, tests were made to determine whether miniature-beta line remained

constant. Females from two culture bottles were used in the experiment and were mated with males homozygous for the gene *S*-1. The parents from both of these cultures were taken from the same mass mating. Eighteen females were tested from each culture. In the test from the first culture all of the females gave miniature males only—indicating that all of the females used in the test had a constant miniature-beta gene. In the test from the second culture, however, sixteen females gave miniature males only but two females had about half of the males miniature and the other half mosaics, indicating that sixteen females were homozygous for the constant miniature-beta gene and two were heterozygous for the constant miniature beta gene and the mutable miniature gene. The data on the male offspring of the two females are given in table 2. The

TABLE 2

THE MALE OFFSPRING OF TWO FEMALES WHICH GAVE MOSAICS

CULTURE NUMBER	MOSAICS	NUMBER OF MINIATURE
10704	46	52
10708	52	62

offspring from the mosaic males of each of these two females were propagated for about fifteen generations and continued to be mutable in somatic tissue in a manner similar to that shown by flies of the miniature-gamma line. Constant miniature-beta gene changed therefore in this case to a mutable miniature-gamma gene.

A small number of mosaic flies which are regularly observed among miniature-beta flies are interpreted as being due to a change from a constant miniature-beta to a mutable miniature. Since all of the mosaic individuals which were tested proved to be constant betas, it is assumed that, in the majority of cases, the change does not affect the germ cells but occurs in somatic cells only.

MINIATURE-GAMMA LINE. The miniature gene of the miniature-gamma line is mutable in the somatic cell but is constant in the germ cells.

Origin.—For many generations the mutable miniature material was propagated by several stock cultures of rather distant common ancestry. In the beginning all mutable miniature material was of the alpha type, viz., it was mutable both in the germ cells and in the somatic cells. During an experiment made with flies taken from a stock line which was propagated by mass matings for about two years it was discovered that the miniature of that line was mutable in somatic cells only. During the time the line was propagated by mass matings, therefore, a change occurred in the miniature-alpha gene making that gene stable in the germ cells but leaving it mutable in the somatic cells; furthermore, during that time, the original alpha gene was entirely lost. Test made later with flies from other stock lines revealed that in all of them miniature-alpha changed into

miniature-gamma form, and for a time the alpha line was entirely lost. It appeared again, however, by a change of a miniature-gamma gene.

Behavior.—The miniature-gamma gene, if no modifiers are present, gives from three to five per cent of mosaic flies. A miniature-gamma line (Demerec, 1928, Table 1[4]) was propagated by pair matings for fourteen generations and during that time produced a total of 275 mosaic and 7855 miniature individuals or 3.4 per cent of mosaics. Another line during sex generations of brother-sister matings produced 121 mosaic and 2426 miniature flies or 4.75 per cent of mosaics. The same line in seven succeeding generations of inbreeding gave only one mosaic fly among 2236 individuals. The tests made at that time indicated, however, that the miniature was not of the gamma type but of the beta type. A change,

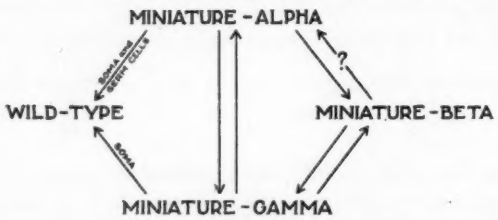

FIGURE 1

Changes which were observed to occur in the mutable miniature gene.

therefore, from gamma to beta had occurred and in the sixth generation of inbreeding beta flies were taken to propagate the line.

As has been shown in previous papers[3,4] the somatic mutability of miniature gamma could be increased by the presence of certain factors. Three such factors are known, two dominant (*S*-1 and *S*-3) and one recessive (*s*-2). Miniature-gamma in the presence of either homozygous or heterozygous *S*-1 or *S*-3 or homozygous *s*-2 produces almost one hundred per cent mosaics. Very few wild-type flies were found in these cases, indicating a high degree of constancy of the miniature-gamma gene in the tissue from which germ cells develop.

DISCUSSION. From the facts presented in this paper it is evident that the changes in the mutable miniature gene occur in several directions, viz., the mutable miniature gene changes to its wild-type allelomorph and also to miniature allelomorphs of different degrees of mutability. As far as can be judged from the tests completed the change to wild type is permanent while the changes to miniature allelomorphs are reversible. In the diagram given in figure 1 the changes are presented which were observed up to the present time. No change from miniature-beta to miniature-alpha has been observed as yet, but only two tests were made by which that change could be detected. It is considered probable that when

a larger number of tests are available the change from miniature-beta to miniature-alpha may be found.

It has been stated before that in an experiment where 72 miniature-alpha genes were tested one change from miniature-alpha to miniature-beta was observed. It has also been mentioned that in an experiment where 36 females were tested, from a miniature-beta line propagated by mass matings for about seventeen generations, two of them were heterozygous for miniature-beta and miniature-gamma. In the seventeenth generation of mass matings, therefore, that line had two gamma genes among 72 genes tested, or that beta line contained 2.78 per cent of gamma-genes. Since the beta line was propagated by random mass matings a change from beta to gamma of a frequency of about 0.17 per cent would be expected to give in the seventeenth generation about 2.78 per cent of gamma flies. The data on hand are too small as yet to compute the frequency with which alpha changed to gamma and beta changed to gamma. It is evident, however, that the frequency of the change for the alpha gene is very much higher, almost ten times the frequency of the change for the beta gene.

In keeping the stocks of the different mutable miniature lines it has been observed that alpha in the stocks changes very frequently to gamma or beta, and that the alpha stock can be kept pure only by a continuous selection. Several changes in the gamma stocks were observed but they were much less frequent than the changes in the alpha stocks. It is possible, therefore, to keep the gamma stock without continuous selection. Changes in the beta stocks are very rare, only two having been observed up to the present time.

The evidence obtained from the keeping of stocks and the results of the experiments in which alpha and beta changed, point to the probability that alpha is the most unstable and beta is the most stable of the three mutable miniature allelomorphs as far as the changes from one miniature allelomorph into another are concerned.

The miniature-gamma character closely duplicates the spotting in mammals by producing heritable mosaics and by having the number and, to a certain extent, the size of mosaic spots modified by several other genes. The mosaics of miniature-gamma are interpreted as being due to the mutability of the miniature gene in somatic cells only. It might be possible that a similar interpretation could explain the behavior of spotting in mammals.

SUMMARY. Three lines of mutable miniature are described, i. e., the alpha line in which the miniature gene is mutable both in the germ cells and in the somatic cells; the gamma line in which the mutability is limited to the somatic cells only, and the beta line in which the gene remains almost constant.

Changes were observed from miniature-alpha to beta and gamma, from miniature-beta to gamma, and from miniature-gamma to alpha and beta.

Miniature-alpha appears to be the most unstable and miniature-beta the most stable of the three mutable miniature allelomorphs as far as the changes from one miniature allelomorph to another are concerned.

[1] Demerec, M., these PROCEEDINGS, **12**, 1926 (687–690).

[2] Miniature-1 is constant allelomorph of mutable miniature and it is independent in origin of mutable miniature. It was found by Dr. C. W. Metz in 1922.

[3] Demerec, M., these PROCEEDINGS, **15**, 1929 (834–838).

[4] Demerec, M., *Verh. V. intern. Kongr. f. Vererbungswis.*, **1**, 1928 (183–193).

CHROMOSOME NUMBER AND THE MUTATION RATE IN AVENA AND TRITICUM[1]

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In common barley (Hordeum vulgare) mutation is readily induced by X-rays.[2] Hundreds of recessive mutations have occurred in this species under known conditions of treatment. The rate of mutation is proportional to radiation intensity, and under heavy dosage of dormant seed mutations have occurred at rates exceeding 5 per cent.

The same treatments applied to common oats yield little or no induced mutation. In 1928 oats were included with barley in a number of experiments in which x-ray treatments were applied to dormant or germinating seeds. Oats of both of the commonly cultivated species, *Avena sativa* and *A. byzantina*, were included, the former species being represented by a selection of the agronomic variety Kherson and the latter by a selection of the variety Fulghum. The rate of mutation was determined for seedling characters only, previous trials having shown that about 90 per cent of the induced mutations were recognizable in the seedling stage. Only one mutation was found in the treated oats, a recessive white seedling segregating in the progeny of Kherson oats irradiated as dormant seed. The same treatments if applied to barley would have produced more than 40 mutations.

This extreme difference in mutation rate between oats and barley is not surprising in view of the known chromosomal constitution of the species concerned. Following the discovery by Sakamura[3] of a polyploid series in Triticum species, similar series have been found in Avena (Kihara[4]) and Hordeum (Griffee[5]). In each genus species with 7, 14 and 21 pairs of chromosomes occur. In *Triticum* and *Avena* the most valuable cul-

tivated species have the highest number of chromosomes, 21 pairs. In *Hordeum*, on the other hand, the cultivated species have 7 pairs of chromosomes, and the higher chromosome numbers are found only in wild species of the genus.

The mode of origin of the species of higher chromosome number is unknown, but it may be assumed that the triple complement of 21 pairs of chromosomes ultimately represents a combination of 3 groups of 7 pairs each, and that these three groups are identical in some of their genes. According to Winge's[6] hypothesis, the higher numbers may have been produced by chromosome doubling in a hybrid between species of lower chromosome number and thus the different groups may be as different as the chromosomes of different species. But species nearly enough related to produce a hybrid must still retain much common germ plasm. Whether the higher chromosome numbers resulted originally from the union of identical or different groups of 7, subsequent mutation and hybridization would lead to further differentiation between the chromosome groups. In either case, therefore, some genes would be identical in all groups and other genes would not; and we have at present no sound basis for estimating the proportion of genes in each class.

There is presumptive evidence of gene reduplication in wheat and oats in the segregation ratios of certain hybrids. Nilsson-Ehle[7] in 1911 reported three genes having the same effect on seed color in common wheat. When a red-seeded plant carrying all three dominant genes is crossed with a white-seeded plant in which all three are recessive, the F_2 ratio is 63 red to 1 white. Some red-seeded varieties have only two of the genes dominant, and these, when crossed with white-seeded plants, give a ratio of 15:1 in F_2. Other varieties, in which only one of the genes is dominant, give a simple 3:1 ratio in F_2. The same situation has been found in the inheritance of several other characters in both wheat and oats, as Gaines[8] has pointed out. Although a segregation ratio of 63:1 does not prove that the three genes concerned are identical, the frequent occurrence of such cases in these species of plants suggests that they contain many genes in triplicate. In barley segregation suggesting duplicate or triplicate genes is rare.

In the case of a dominant gene homozygous in all three groups, the recessive mutant character would not appear unless recessive mutation occurred coincidentally at the same locus in the three groups. For example, let A represent a dominant gene essential to chlorophyll development, A_1, A_2 and A_3 representing this gene in the three chromosome groups. A normal plant of common barley is $\frac{A}{A}$. Mutation of A to a produces the heterozygote $\frac{A}{a}$ and in the next generation this segregates 25% albino

plants, the homozygous recessive $\frac{a}{a}$. But a normal plant of common oats may be $\frac{A_1}{A_1}\frac{A_2}{A_2}\frac{A_3}{A_3}$. Mutation of A_1 to a_1 produces the heterozygote $\frac{A_1}{a_1}\frac{A_2}{A_2}\frac{A_3}{A_3}$, which by segregation produces the homozygous recessive $\frac{a_1}{a_1}\frac{A_2}{A_2}\frac{A_3}{A_3}$. This plant is green because of A_2 and A_3, and since A_2 and A_3 are homozygous, it will (barring further mutation) produce only green descendants. A segregation of albino plants from the homozygous triple dominant could be produced only by inducing mutation in A_1, A_2 and A_3 in the same cell. If mutations were induced simultaneously in one allelomorph of each pair, the recessive mutant character would appear in $^1/_{64}$ of the progeny. Similarly, in a plant in which the gene is dominant in two of the three groups $\left(\frac{A_1}{A_1}\frac{A_2}{A_2}\frac{a_3}{a_3}\right)$, simultaneous mutation of A_1 and A_2 would be required for segregation of the recessive character, which would then appear in $^1/_{16}$ of the progeny. The probability of occurrence of these coincidences is remote. Recessive mutations of appreciable frequency are to be expected only in those genes which are dominant in only one group $\left(\frac{A_1}{A_1}\frac{a_2}{a_2}\frac{a_3}{a_3}\right)$.

If gene reduplication is the cause of the low apparent mutation rate in oats, common wheat also should yield few mutations under x-ray treatment, and in the species of both *Avena* and *Triticum* it should be possible to show a relation between chromosome number and mutation rate. To test this relation the mutation rate under x-ray treatment was determined in species of varying chromosome number in both genera.

The experimental technique was similar to that previously described for barley,[2,9] except in certain details here stated. Unfiltered radiation from a Universal type Coolidge x-ray tube operated at 108 K. V. P., 4 m. a. tube current, 18 cm. target distance, was used for periods of 5, 10, 20, 30 or 40 minutes. The intensity of the radiation emitted under these conditions (measured in international *r*-units by means of a Victoreen dosimeter) is 290 *r* per minute of exposure. Dormant seeds were treated in all cases. The seeds were planted immediately after treatment, under conditions favorable to tillering. Seedling progenies were grown in the greenhouse from the main head and the first two tiller heads of each plant, later tillers being discarded because they are in some cases derived from earlier tillers. Mutation rates were determined from the progeny of tillering plants only, and in all cases recorded as mutations the mutant character segregated in the progeny of one head and was absent in other head progenies of the same plant. The primordia of the three heads tested are separate in the embryo at the time of treatment, and the frequency of mutation is indicated by the number of seedling mutations found in pro-

portion to the number of head progenies tested. Previous trials[10] with varying doses of x-rays applied to dormant seed of barley have shown that the mutation rate per unit of radiation intensity is a constant. This constant is a convenient basis of comparison of mutation frequency in different species.

The frequency of mutation in *Hordeum vulgare* under these conditions was determined for comparison with the species of *Avena* and *Triticum*. Dormant seed of the agronomic variety Himalaya was irradiated for periods of 10, 20, 30 and 40 minutes. The mutation rate per *r*-unit was $(4.9 \pm 0.9) \times 10^{-6}$.

Four species of *Avena*, *A. brevis*, *A. strigosa*, *A. byzantina*, and *A. sativa*, were treated. Kihara in 1919 reported 7 pairs of chromosomes in *A. strigosa*, and later 7 pairs were reported in both *A. brevis* and *A. strigosa* by Huskins,[11] Goulden,[12] and Aase and Powers.[13] *A. byzantina* and *A. sativa* are the common oats species with 21 pairs of chromosomes in which trials had been made in the previous season.

Seed of all four species was irradiated for periods of 10, 20, 30 and 40 minutes. The plants of *A. brevis* and *A. strigosa* were distinctly injured in early growth by the 30-minute and 40-minute exposures, but recovered sufficiently to produce some tillering plants suitable for the determination of mutation rate from all treatments. The injury to the common oat species was slight in early growth and inappreciable at maturity. An additional and more extensive trial was made in *A. brevis* and *A. strigosa* irradiated for 5 minutes, and in *A. strigosa* irradiated for 10 minutes. The frequency of mutation in the four species is shown in table 1.

TABLE 1

FREQUENCY OF INDUCED MUTATION IN SPECIES OF AVENA

SPECIES	CHROMOSOME NUMBER	MUTATION FREQUENCY DURATION OF TREATMENT (MIN.) 5	10	20	30	40	MUTATION RATE PER *r*-UNIT ($\times 10^{-6}$)
A. brevis	7_{II}	0/268	0/24	2/51	3/39	0/12	4.1 ± 1.2
A. strigosa	7_{II}	2/449	3/484	1/101	1/44	2/38	2.6 ± 0.6
A. byzantina	21_{II}		0/116	0/102	0/64	0/55	0
A. sativa	21_{II}		0/133	0/124	0/101	0/55	0

A. byzantina and *A. sativa*, the species with 21 chromosome pairs, yielded no mutations. The treatments applied to these species in this trial, together with those applied in the previous season, were sufficient to have produced more than 70 mutations in barley, instead of the single mutation found in oats.

The two 7-chromosome species of *Avena* yielded 14 mutations. Although the mutation rates in both of these species were somewhat lower than that of barley, the differences are not statistically significant. The mutant characters in *A. brevis* were "white" (3 cases), "virescent," and "pale yellow;" in *A. strigosa* "white" (2 cases), "pale green" (2 cases), "greenish yellow" (2 cases), "virescent," "banded," and "brownish."

All of these types except "brownish" are similar to mutant types previously found in barley.

The species of *Triticum* treated were *T. monococcum* (7 pairs of chromosomes), *T. durum* and *T. dicoccum* (14 pairs), and *T. vulgare* (21 pairs). The chromosome numbers given have been reported by Sakamura,[3] Kihara,[4] Sax,[14] and others. In the fall of 1928, 30-minute and 40-minute treatments were applied to *T. sativum* of the fall-sown variety Harvest Queen. In the following spring, treatments of 10, 20, 30 and 40 minutes were applied to spring-sown varieties of all four species, of which seed was kindly furnished by Professor H. K. Hayes, of the University of Minnesota. *T. monococcum* was so severely injured by exposures of 20 minutes or more that mutation rates could not be determined from any but the 10-minute treatment. *T. dicoccum* also was severely injured by the 20-minute and 30-minute exposures, but matured a few tillering plants given these treatments. Neither of these species matured seed from the 40-minute treatment. *T. vulgare* was injured by the 30-minute and 40-minute exposure, but recovered sufficiently to permit the determination of mutation rates from a small number of plants. *T. durum* was not seriously injured by any of the treatments used.

The frequency of mutation in the species of Triticum is shown in Table 2.

TABLE 2

FREQUENCY OF INDUCED MUTATION IN SPECIES OF TRITICUM

SPECIES	CHROMOSOME NUMBER	MUTATION FREQUENCY DURATION OF TREATMENT (MIN.) 10	20	30	40	MUTATION RATE PER r-UNIT ($\times 10^{-6}$)
T. monococcum	7_{II}	4/133				10.4 ± 3.4
T. dicoccum	14_{II}	0/61	1/26	0/20		2.0 ± 1.3
T. durum	14_{II}	0/55	3/237	1/73	2/79	1.9 ± 0.5
T. vulgare	21_{II}	0/52	0/17	0/377	0/299	0

T. monococcum, though tested only on a small scale, yielded four mutations. Its mutation rate per unit of radiation intensity was higher than that of barley, though the difference is not statistically significant. The mutant types were "white" (2 cases), "virescent," and "yellowish-green."

The two species with the doubled number of chromosomes mutated at a lower rate. The test of *T. dicoccum* was small and only one mutation was found. The mutant type was "white." In the more extensive trial with *T. durum* 6 mutations were found. The mutant types were "yellow" (3 cases), "light yellow," "white," and "shriveled." In addition a white seedling segregation was found in the progeny of an untillered plant. All of the segregation ratios were apparently monogenic, like those from the 7-chromosome species, and probably all were due to the mutation of a gene dominant in only one group.

T. vulgare, though much more extensively tested, yielded no mutations. The treatments applied were sufficient to have given about 40 mutations in barley.

The results as a whole support the hypothesis that the frequency of induced mutation observed in polyploid species is low because of gene reduplication. This does not imply that other factors may not affect mutation rate as well. It is possible that the basic mutation rate may differ materially in different groups. The data available in *Avena* and *Triticum* are not yet sufficiently extensive to permit strict comparison between individual species. But it is clear in both genera that the mutation frequency (as measured by visible effects) decreases sharply with increasing chromosome number. Apparently the proportion of mutable genes not reduplicated is very small in the species with the triple chromosome number. In the species with the double chromosome number a considerable proportion of the mutable genes appear to be dominant in only one of the two chromosome groups.

In untreated plants the rate of change due to recessive mutation is doubtless decreased similarly by polyploidy. Probably this is the reason for the fact that seedling chlorophyll characters, though of fairly common occurrence as recessives in barley, have been found only rarely in wheat and oats. These characters are relatively inviable or lethal, and in regularly self-fertilized species their causal genes would quickly be eliminated from the germ plasm. The deleterious characters found are probably due to mutations of relatively recent occurrence, and these apparently have been much less frequent in wheat and oats than in barley.

The expression of dominant mutation, on the contrary, would not be affected by polyploidy. Three-fourths of the progeny of the mutating plant would have the new dominant gene in one of the chromosome groups, and these would show the dominant character. In fact, the frequency of dominant mutation would probably be somewhat higher in the polyploid species, since the number of genes which might mutate would be increased. But if the mutations induced by x-rays are representative of mutation under natural conditions, the proportion of dominant mutations is extremely small.

[1] These experiments were supported in part by a grant from the NATIONAL RESEARCH COUNCIL, Committee on the Effects of Radiation upon Living Organisms.

[2] Stadler, L. J., *Science*, n. s., **68**, 186–187 (1928).

[3] Sakamura, T., *Bot. Mag.*, **32**, 151–154 (1918).

[4] Kihara, H., *Bot. Mag.*, **35**, 19–44 (1919).

[5] Griffee, Fred, *Univ. Minn. Studies Biol. Sci.*, **6**, 319–331 (1927).

[6] Winge, Ö., *Compt. Rend. Trav. Lab.*, Carlsberg, **13**, 131–275 (1917).

[7] Nilsson-Ehle, H., *Lunds Univ. Arsskr. N. F.*, Afd 2, Bd. 7, Nr. 6, 3–84 (1911).

[8] Gaines, E. F., *J. Amer. Soc. Agron.*, **19**, 202–205 (1927).

[9] Stadler, L. J., *J. Hered.*, (1929) (in press).

[10] Stadler, L. J., *Anat. Rec.*, **41**, 97 (1928).

[11] Huskins, C. L., *Sci. Agr.*, **6**, 303–313 (1926).

[12] Goulden, C. H., *Minn. Agr. Expt. Sta. Tech. Bul.*, **33** (1926).

[13] Aase, Hannah C., and Le Roy Powers, *Am. J. Bot.*, **13**, 367–372 (1926).

[14] Sax, K., *Science*, n. s., **54**, 413–416 (1921).

A PROPOSED PHYLOGENY OF THE ONAGRACEAE BASED PRIMARILY ON NUMBER OF CHROMOSOMES

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The family Onagraceae is such a homogenous one that the interrelationships of the constituent genera should not be difficult to determine. Moreover, as is well known, changes in number of chromosomes play an unusually conspicuous rôle in experimental phylogenetic studies in this family.

There is no basic number of chromosomes common to all genera in the family, which indicates that the family probably originated from parents of dissimilar chromosomal constitution.[1] Postulating a "hybrid" ancestry renders it possible to consider at least five distinct sets of haploid chromosome numbers upon which to base a phylogenetic scheme. If the structure were based upon a single ancestor whose parents each had essentially the identical chromosomal complements, only one haploid set is available; in other words, the possible mechanical combinations are strictly limited and hardly any flexibility is possible unless polyploidy of some form intervenes, or it is assumed that the ancestor showed marked variations in the number of chromosomes.[2]

If we visualize one parent as an aquatic form possessing four haploid chromosomes, and the other as a probably terrestrial form with seven haploid elements, the cross between the two would therefore possess eleven haploid chromosomes. We may call the first parent A, the second B, and the hybrid C. We may assume that A and B are self-fertile and that C is capable of being back-crossed to either parent. Therefore, $A + C = 15$, and $B + C = 18n$ chromosomes. All numbers so far found for any onagrad, with a few improbable exceptions,[3] can be fitted nicely into this scheme.

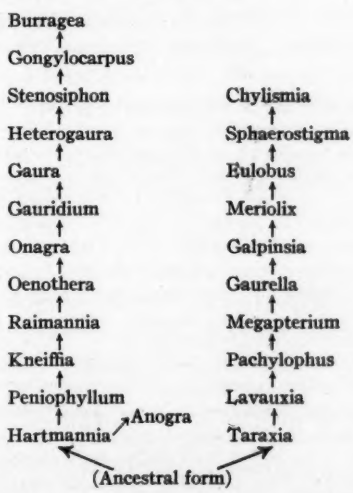
Before proceeding further, it will be necessary to mention that the chromosomal complement of any species in the following genera has not yet been ascertained: Hauya, Gayophytum, Meriolix, Diplandra, Riesenbachia, Semeiandra, Ludwigia, Heterogaura, Gongylocarpus, Burragea. Some of these have been fitted into the scheme on the basis of vegetative and morphological resemblances.

Jussieua (Jussiaea), with $8n$ chromosomes, may be assumed to have arisen directly from A as a tetraploid genus. Parent A probably had many of the characters of Ludwigia, and there is a strong likelihood that Jussieua may have been derived from this genus. The back-cross of A with C would account for the parentage of Zauschneria. On the basis of its

vegetative characters as well as in the number of chromosomes ($15n$) Zauschneria cannot readily be associated with any other onagrad, hence the independent origin of this genus is all the more plausible. Hauya is conceived as being closely connected to Zauschneria.

Much the larger number of genera may be arranged in two distinct series which may be traced back to parent *B*. The first, though not necessarily the more primitive, group begins with Boisduvalia and progresses to Godetia, thence to Clarkia and ends in Eucharidium. All four genera possess $7n$ chromosomes, more or less, the variations being due to fragmentation, fusion and polysomy.

It is highly probable that the ancestral form of the other group, which might be called the Oenothera-stem, is not now recognizable. From this assumed progenitor two distinct lines may be traced, and all genera in which the basic number of chromosomes has been ascertained possess $7n$ elements. A diagrammatic scheme will bring out the relationships of the entire group more clearly; this outline is primarily intended to indicate relationships rather than actual lines of descent:



Gauridium provides a most convincing connecting link between the Oenothera and Gaura groups; this was recognized as long ago as 1797 by Curtis.[4]

The back-crossing of *C* with *B* doubtless resulted in our present Epilobiums, in which $18n$ chromosomes are found with remarkable constancy. Gayophytum is probably closer to Epilobium than has been generally recognized, having over 20 diploid chromosomes; I have been unable, on

account of paucity of material and the extremely small size of the chromosomes, to determine the number accurately.

The hybrid parental form, *C*, may be conceived as the immediate ancestor of Circaea;[5] that genus may even include the hybrid form itself. From this genus the line proceeds directly to Lopezia, thence to Fuchsia and to the two genera dubiously segregated from Fuchsia, Encliandra and Skinnera: in all these genera the basic haploid number is 11. In their relationships to Fuchsia, the latter two genera occupy a position analogous to that occupied by Eucharidium and Clarkia in relation to Godetia. On the basis of vegetative resemblance, Riesenbachia, Diplandra and Semeiandra probably descended from Lopezia, but their exact position should be considered somewhat doubtful pending further investigation.

Gauropsis, Ludwigiantha, Ludwigiaria and Xylonagra are insufficiently understood, while the remaining genera are considered synonyms: Agassizia, Allochroa, Baumannia, Blennoderma, Brebissonia, Burmannia, Calylophus, Chamaenerion,[6] Chamissonia,[7] Cratericarpium, Crossostigma, Cubospermum, Dantia, Dictyopetalum, Dorvalia, Heterostemon, Holostigma, Isnardia,[8] Jehlia, Kierschlegeria, Lyciopsis, Lysimachion, Nahusia, Nematopyxis, Onosuris, Oöcarpon, Opsianthus, Pachydium, Phaeostoma, Pleurandra, Pieurostemon, Prieurea, Quelusia, Schizocarya, Schufia, Xylopleurum.

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[1] Earlier lists of chromosome numbers may be found in Gaiser, *Genetica*, **8**, 432–434 (1926); Tischler, *Tabulae Biologicae*, **4**, 39–41 (1927). The writer has made original counts for the following species (except where noted, the haploid number is the one included in parentheses): *Anogra trichocalyx* (7); *Boisduvalia glabella* var. *campestris* (7); *Chylismia clavaeformis* (7); *Circaea pacifica* (11); *Clarkia elegans* (n = 3–13 inclusive; $2n$ = 7–22 inclusive), *C. pulchella* (n = 4–14; $2n$ = 8–16), *C. rhomboidea* (7, 9); *Epilobium obcordatum* (18), *E. californicum* (18), *E. paniculatum* and the variety *jucundum* (18), *E. watsoni* var. *franciscanum* (18); *Galpinsia hartwegi* (7); *Gaura coccinea* (7); *G. lindheimeri* ($2n$ = 14); *Gauridium molle* (7); *Godetia quadrivulnera* ($2n$ = 14); *G. amoena* and the variety *lindleyi* (7); *G. deflexa* (ca. 9); *Fuchsia magellanica* var. *riccartoni* and var. *gracilis* (11); *Hartmannia tetraptera* (7); *Megapterium missouriense* (7); *Sphaerostigma veitchianum* (7); *S. spirale* (7); *S. dentatum* and the variety *campestre* (7); *Stenosiphon linifolium* (7); *Taraxia heteranthera* var. *taraxacifolia* ($2n$ = ca. 14); *T. ovata* (7); *Zauschneria californica* (several forms with 7, 8, 15 and 30), *Z. canescens* ($2n$ = ca. 30); *Z. microphylla* ($2n$ = ca. 30). In addition, counts for the following species have been confirmed: *Epilobium angustifolium* (18); *Eucharidium concinnum* (7); *Skinnera procumbens* (11); *Onagra hookeri* (7). See also Johansen, *Amer. J. Bot.*, **16**, 595–597, 1929.

[2] Broekens (*Rec. trav. bot. Néerl.*, **21**, 383–512 (1924)) did not pay much attention to number of chromosomes in erecting his scheme, yet there are many points where his scheme agrees with the one here presented. The principal difference is that slightly more emphasis has been placed upon monotypic and very small genera in my outline.

[3] There may be certain species whose chromosomal constitution is open to doubt; for example, the count of $9n$ chromosomes reported for *Godetia bottae* was made on cultivated plants and is therefore in need of confirmation. However, I find that *G.*

deflexa, separated from *G. bottae* by Jepson, also has about 9*n* elements, as noted above.

[4] Curtis, *Curtis' Bot. Mag.*, **11**, 388, 389, 1797.

[5] My counts were made on *Circaea pacifica* in 1926; see also Uddling, *Hereditas*, **12**, 294–296, 1929.

[6] I can find no substantial cytological characters differentiating Chamaenerion from Epilobium.

[7] Synonym of Sphaerostigma: consult Munz, *Bot. Gaz.*, **85**, 233–270, 1928. The action of Munz in reducing Sphaerostigma, Taraxia, Eucharidium and other genera to subgeneric rank, I consider unfortunate and not warranted by the morphological criteria.

[8] May possibly be cytologically distinct from Ludwigia, but I have no cytological acquaintance with any species placed under Isnardia.

THE CHROMOSOMES OF BUGINVILLAEA

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A study is being made of the chromosomes in the pollen mother cells of *Buginvillaea glabra* Choisy. Division figures are readily obtained in flower buds that are from 3 to 4 mm. in length. When the equatorial plates of the heterotypic division were first studied it was found that the chromosomes of one pair were sufficiently different in size to be mistaken for sex chromosomes. Since this plant produces perfect flowers, the unequal distribution of chromatin can certainly have no connection with sex-differentiation.

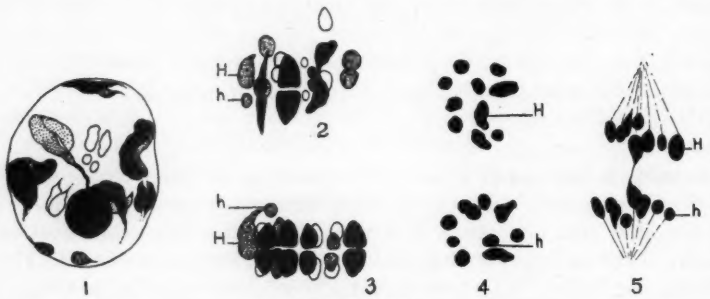
Material for comparative study has been obtained from various sources. During the year 1927–28 young buds were collected from plants of *B. glabra* which have been growing in the greenhouses at Purdue University for many years. While on a vacation in Florida during the winter of 1928–29, Miss Ruth Lindsay made fixations of material from plants growing out of doors. Fixations have also been made of the horticultural variety Crimson Lake. The chromosomes of all these plants show a precisely similar unequal pair.

The majority of the material for this study was collected from plants growing in the greenhouses at the University of Wisconsin. These plants of *B. glabra*, according to the records available, originally came from Bermuda. Specimens were sent to Dr. J. M. Greenman of the Missouri Botanical Garden, who identified them as *Buginvillaea glabra*. The plants of Crimson Lake were obtained from the Royal Palm Nursery, Oneca, Florida, and grown in the greenhouses at the University of Wisconsin. Dr. L. H. Bailey considers this to be a form of *Buginvillaea spectabilis* Willd.

On the basis of all information thus far obtained, both *B. glabra* and *B.*

spectabilis are self-sterile under the conditions found in Florida, the Canal Zone, and the British West Indies. I have been unable to obtain seeds of these species from any source. Information from Trinidad and Jamaica indicates that, when these species are crossed, seeds may be formed to a very limited extent. I have made approximately fifty attempts at cross-pollination between flowers of the same and of the two different forms and numerous attempts to self-pollinate the flowers, but without success. The pollinated flowers wither quickly and drop, just as do those which have not been touched.

The stages of diakinesis, metaphases, and early anaphases of the heterotypic division in the pollen mother cells are very favorable for observing the chromosomes. The bivalent nature of the chromosome pairs is clearly



FIGURES 1–5

Fig. 1. Diakinesis (*Buginvillaea glabra*). Fig. 2. Equatorial plate (*B. glabra*), showing the unequal pair of chromosomes, *H* and *h*. Fig. 3. Equatorial plate (Crimson Lake). Fig. 4. The two chromosome groups seen in a polar view of a heterotypic anaphase (*B. glabra*). Fig. 5. Early heterotypic anaphase (*B. glabra*).

evident in diakinesis (figure 1), and ten pairs can easily be distinguished. At this stage there are considerable differences in size between the chromosome pairs.

On the heterotypic equatorial plate the chromosomes are closely grouped so that it is somewhat difficult to count them in lateral view (figures 2 and 3). As they separate, the attachment of the spindle fibers is seen to be telomitic. The chromosomes are somewhat rounded, and constant differences in size appear between the respective pairs. Two pairs are very small, four pairs are intermediate in size, and three pairs are relatively large. Among the three large pairs, one is usually conspicuous in that the chromosomes adhere to each other as they move toward the poles and are drawn out into a dumb-bell shape before splitting. The tenth pair consists of a large (*H*) and a small (*h*) chromosome. This heteromorphic pair separates so that the large chromosome goes to one pole and the small one to the other.

The early anaphases (figure 5) show the separation of the chromosomes of this heteromorphic pair. When the anaphase chromosomes are seen in polar view (figure 4), they are similarly arranged in the two groups on the same spindle. In one group the large chromosome (*H*) is found in the same relative position as the small one (*h*) in the other group. In the homeotypic division the chromosomes are so closely packed together that it is difficult to make an accurate count, and still more so to make a comparison of relative sizes. The homeotypic division seems to be regular, and typical pollen tetrads are formed. At the time of dispersal the pollen grains appear to be normal.

The observations of unequal homologous chromosomes in plants thus far reported are in dioecious species in which the inequality of the chromosomes of one pair could or might be associated with sexual differentiation. Miss Carothers,[1] however, has found heteromorphic pairs of homologous chromosomes distinct from the sex chromosome in both male and female individuals of several orthoptera.

When it was found that these species of *Buginvillaea* do not produce seeds, an attempt was made to germinate the pollen grains in order to determine the percentage, if any, of viable grains. Pollen from some 200 flowers was placed in sugar solutions of concentrations varying from 2% to 20%. The pollen was examined at the end of 24 hours and again at the end of the second day, and no germination had taken place. The introduction of a portion of the stigma into the drop of sugar solution did not stimulate germination.

[1] Carothers, E. E., *J. Morph.*, **24**, 487–511 (1913); **28**, 445–494 (1916–1917); **35**, 457–474 (1921).

NEW RESULTS FROM THE STUDY OF COELENTERATE NEMATOCYSTS
(Preliminary Note)

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Through the aid of a Fellowship of the *International Education Board* I have been enabled to continue, under favorable conditions and on material of particular interest, my researches on the nematocysts of Coelenterates, which I have already studied for several years. After a study, chiefly physiological, I had recognized the importance and the necessity of a systematic morphological investigation of these organites in the various groups of this phylum; and as this required living material, I particularly appreciated the opportunity of collecting and studying non-European

species also, many of which seemed to be indispensable for forming a general idea of the evolution of these strange structures.

Through this aid I have been able to collect extremely interesting material at widely separated stations: at the *Marine Biological Station* of Nhatrang-Cauda (Oceanographic Service of Fisheries), Annam, French Indo-China (February–May, 1928); at the *Marine Biological Laboratory*, Woods Hole, Mass. (July–September, 1928); at the *Hopkins Marine Station*, Pacific Grove, Cal. (November, 1928–January, 1929); and lastly at the *Bermuda Biological Station for Research* and the *Bermuda Government Aquarium* (January–April, 1929). The material was of particular interest and abundance especially in Indo-China and in Bermuda, owing to the classical wealth and the fascinating variety of the coral reefs. I am greatly indebted to the Directors of these laboratories, Dr. A. Krempf, Prof. M. H. Jacobs, Prof. W. K. Fisher, Prof. E. L. Mark, and Mr. L. L. Mowbray, and their staffs, for their kind hospitality and for the many facilities they let me have, without which this work could not have been done.

During this study 52 species, previously unknown to me, have been investigated, thus raising to 109 the total number of species I have examined. The list of these species, the technique which has been used, and the results obtained, will be fully reported in an extensive monograph now under way. Yet it may be useful to give here a short preliminary account of the main results obtained during this last year, embracing the data establishing some new ideas, but not those which merely confirm observations made previously by other authors.

I. MORPHOLOGY OF THE NEMATOCYST.—As a general result of my observations there have been established nineteen types of nematocysts characterized by the structure of the everted thread. These nineteen types constitute an almost uninterrupted series, ramified in several parallel directions: there are six fundamental types, characterized by the more or less complicated shape of the everted thread, each of which leads, through identical variations in the armature of the thread, to several highly and differently specialized forms. This series shows all the features interpreted as Orthogenesis and Adaptation in the comparative study of the living species, but opens here a problem of particular interest and complexity: this series is not autonomous, but is only part of, though causally connected with, the evolutionary line pursued by the Coelenterates themselves. So that here we have to deal with two independent, but simultaneous, evolutionary lines, showing the same characteristics (orientation, orthogenesis, adaptation), the extent of parallelism between which is to be investigated. The question is still further complicated by the fact that, first, most of the Cnidaria bear simultaneously several types of nematocysts, representing several stages of nematocyst evolution, within the

same animal, and, secondly, that the same animal can bear different cnidoms (1) in the various stages of its life-history. It would be out of place to discuss here all the data connected with this question, but it appears from a study of them that the two evolutionary lines (Coelenterate, nematocyst) are not always parallel, and that in the case of the nematocysts, as in many others, the interpretations of Orthogenesis and Adaptation originate merely from a subjective fallacy.

From a more technical point of view, it has been possible to make a first attempt at a general classification of nematocysts, and to furnish an appropriate terminology; it is believed that this classification and terminology will have the same usefulness in the study of the Cnidaria as have those of the spicules in the study of the Sponges.

II. TAXONOMIC VALUE OF THE NEMATOCYST.—The composition of the cnidom has been established with accuracy for all the collected species. Its taxonomic value, which theoretical considerations permitted one to affirm beforehand, has been corroborated by the facts. In a large number of cases, the nematocysts have allowed me to foretell anatomical or embryological features of the Coelenterate exhibiting them, which subsequently have been confirmed when these features were studied. For instance, it has been possible, through the study of the nematocysts alone, to conclude that the incompletely known *Clava krempfi* Billard (2) is very different from the true *Clava*, and to foretell that its gonozooid must be a free medusa, belonging to the family Pteronemidae. This conclusion has been confirmed (see under VI, c).

In a certain number of cases the relationships thus demonstrated do not agree with the ideas hitherto advocated. Particularly in the classification of the Leptolida, the many resulting contradictions and incompatibilities permitted one to foresee that the characters usually considered important were here of little or no value. The nematocysts, however, always gave precise and concordant indications, which therefore seem to show more adequately the natural relationships of the species.

So far as nematocysts indicate relationships:

(a) The Hydrides are not the simplest, but among the most complicated Hydrozoa; they are closely connected with the Tubularians.

(b) The Calyptoblasts/Leptomedusae are a rather homogeneous group; on the contrary the Gymnoblasts/Anthomedusae include very dissimilar forms, and it will be impossible to maintain this group. To be removed are particularly the (medusae) Pteronemidae, usually budded from *Coryne*-like polyps, and the gen. *Eudendrium*, the relationships of which are, in fact, with the Calyptoblasts.

(c) The Siphonophora include at least two different groups, as already stated by Haeckel on other grounds and in a different interpretation; one group (Siphonantha) is related with the true "Anthomedusae," re-

stricted as stated above, the other (Disconantha) is to be connected rather with the Scyphomedusae.

(d) The Milleporida constitute a homogeneous group, not related to the "Gymnoblasts" s. str., but perhaps to the Pteronemidae.

(e) The Trachylida (Tracho- and Narcomedusidae) constitute a homogeneous, probably primitive, group (3), the affinities of which are doubtful.

(f) The "spirocysts" (4, 5), supposed to be characteristic of all the Anthozoa, are lacking, in fact, in all the Octocorallia; in certain Hexactinidae (*Actinotryx*, *Lebrunia*, *Zoanthus*) a quantitative reduction of these structures (spirocysts) has been observed, going almost as far as their complete disappearance.

(g) The Hexacorallia constitute a well-delimited group, separated from the Actiniaria. The Stichodactylina, considered to be a homogeneous group, include, in fact, two categories of species, one belong to the Hexacorallia, the other to the Actiniaria; an identical conclusion had been reached by Krempf (6) through the study of their anatomical and embryological features.

(h) The gen. *Lebrunia* on the one hand, and the family Sagartiadae on the other, bear each a special kind of nematocyst not found anywhere else.

The observations here recorded have been extended, it is true, to only a very few species, representing a very small part of this large phylum. Yet, they have been so concordant, that it seems worth while to state them, without prejudging, of course, the degree of generality they will be found later to deserve. At best, conclusions of this kind can never be definitive; their purpose can be only to account for the present state of our knowledge, to connect the observed facts, and to prepare the field for further researches.

III. PHYSIOLOGY OF THE NEMATOCYST.—(a) In the large nematocysts with isodiametrical and regularly armed tube, found in the Hexacorallia (see II, g), the capsular content, stained with neutral-red, is soluble in seawater, while in the other kinds of nematocysts it is insoluble. Thus there exists here a relation between the structure of the tube and the physicochemical properties of the capsular content.

(b) However, contrary to what would have been expected according to the theories of Adaptation, there is not always a fixed relation between the structure of the nematocyst and its toxicity: e. g., in the nematocyst of the tentacles of *Physalia arethusa* Brown, well known for their great power of urtication, the armature of spines is almost entirely lacking, and the everted thread is incapable of penetrating into any tissue; the poison must be effective by simple contact. In other cases, strongly armed nematocysts are destitute of any chemical efficiency, and are sometimes located in tissues and organs far away from the surface of the body. In still other

forms of nematocysts, the tube is closed at its tip, or prevented, by accessory structures, from completely everting itself. All these facts are against a too adaptavistic interpretation of these structures and of the enigmatic forces out of which they originated.

(c) The nematocysts found in the mesenteries of *Actinotryx sanctithomae* Duch. & Mich. and of *A. macropapillata n. sp.* (see VI, b) provide a most favorable object for physiological research; their perfectly regular shape is convenient for volumetric measurements; their exceptional size (up to 70 × 250μ, with a thread attaining a length of 6 mm.!) easily permits manipulation of isolated capsules and experimentation with them.

A nematocyst can be isolated in a small drop of water; the thread-eversion can then be produced by pressure on the cover-glass and the everting thread directed in such a way as finally to cause its tip and the final ejaculation of the capsular content to lie outside of the drop; thus it is possible to measure the surface occupied by the capsular content after its spreading out on the slide, and, its volume having been measured previously, to calculate approximately its viscosity and thus the amount of energy necessitated for the eversion of the thread.

(d) It is possible to measure the variations in volume resulting from (or perhaps responsible for) the thread-eversion: the volume of the capsule decreases, but the total volume of the nematocyst (i.e., capsule plus everted tube) increases with the thread-eversion. The length of the capsule decreases twice as much as its width.

IV. DEVELOPMENT OF THE NEMATOCYST.—(a) In a still undetermined species of Siphonophore (collected at Pacific Grove) the development of the nematocyst begins remote from the tentacular knobs, within the manubrium of a peculiar kind of medusozooid, resembling the male gonozooid, which apparently has not yet been described.

(b) In *Tubularia marina* Torrey, contrary to what had been observed in three other species of the same genus (1), the "specific" nematocysts of the adult do not result from the transformation of the "larval" ones.

(c) The polyp of "*Clava*" *Krempfi* Billard bears two categories of nematocysts; only one of them migrates into the medusa (see VI, c), which, in addition, develops a third kind.

V. COMPARATIVE ANATOMY OF THE ANTHOZOA.—The study of the nematocysts shows that the marginal tubercles of *Bunodes* are homologous with the "cnidorrhags" of *Actinia equina* L.; but the wall-tubercles of *Bunodactis* are to be considered merely as ectodermic proliferations, not homologous with the former.

VI. FAUNISTIC NOTES.—(a) Through the study of the cnidom it is possible to separate the sp. *Alcyonium krempfi* Hickson (7) into two forms (varieties[2]).

(b) In Bermuda has been found a new Actinian, belonging to the gen.

Actinotryx, and characterized chiefly by the location and the remarkable size of its disc tubercles.

(c) In Indo-China has been observed the sexual phase of the Gymnoblastic Hydroid *Clava krempfi* Billard (2); it consists in a free medusa belonging to the family Pteronemidae. All other species of this family are known to be budded by *Coryne*-like polyps; therefore a new genus has to be created for this species.

(d) Concerning the parasites of Coelenterates, I have found, in some Indo-Chinese Actinians, an Infusorian, *Foettingeria actiniarum* Caull. & Mesn., observed hitherto only in European seas, and in an Alcyonidian (*Nephthya* spec.), a Microsporidium, which is the first of the Cnidosporidia to be discovered in a Cnidarian.

* INTERNATIONAL EDUCATION BOARD FELLOW.

[1] Weill, R. F., *Bull. Soc. Zoöl. France*, **51**, 1926 (254–260).
[2] Billard, A., *Bull. Mus. Hist. Nat., Paris*, **25**, 1919 (187–188).
[3] Weill, R. F., *C. R. Acad. Sc., Paris*, **182**, 1926 (1357–1359).
[4] Bedot, M., *Rev. Suisse Zoöl. Genève*, **3** [4], 1895 (533–539).
[5] Will, L., *Sitzungsb. Abh. Nat. Ges. Rostock*, **1**, 1909 (65–103).
[6] Krempf, A., *C. R. Acad. Sc., Paris*, **139**, 1904 (806–810).
[7] Hickson, S. J., *Bull. Soc. Zoöl. France*, **44**, 1919 (411–424).

AN INTERFEROMETER METHOD OF OBSERVING THE VIBRATIONS OF AN OSCILLATING QUARTZ PLATE

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The work here reported involves a somewhat novel use of the Michelson interferometer in the investigation of the modes of vibration of an oscillating quartz crystal. One interferometer mirror is replaced by the crystal which is so mounted that reflection occurs at the surface to be examined. If parallel fringes are produced as usual by tilting one of the reflecting surfaces, a vibration of the quartz surface will produce a disturbance of the fringe system. The straight fringes are localized; hence from the variation in the appearance of the fringes from point to point, one can draw conclusions as to the corresponding variation of amplitude of vibration of the quartz surface.

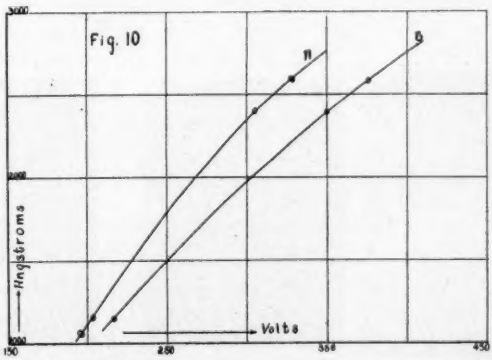
The effect of a normal oscillation of a surface element of the plate will be to impart lateral oscillations to the corresponding fringe system. The brightness of the non-oscillating fringes is given by the equation

$$y = K\left(1 + \cos\frac{2\pi X}{h}\right), \qquad (1)$$

where K is a constant, h is the fringe width, and X is the perpendicular distance across the fringes measured from any convenient reference point. In view of the rapid oscillations of the crystal, the effective brightness at a point X in the oscillating fringe system is the time average of the instantaneous brightness at X. This effective brightness I, where the crystal is assumed to oscillate with simple harmonic motion, is given by the formula

$$I = K\left(1 + J_0\left(\frac{4\pi r}{\lambda}\right) \cdot \cos\frac{2\pi X}{h}\right), \qquad (2)$$

where r is the amplitude of the simple harmonic motion of the surface element, λ is the wave-length of light employed, and J_0 is a Bessel function of



zero order. In what follows the fringes produced by a surface element at rest will be denoted as y fringes; the fringes above, as I fringes.

J_0 is equal to unity when r is zero. Thus, as the amplitude of the simple harmonic motion approaches zero, I reduces to y. As r is increased, $J_0\left(\frac{4\pi r}{\lambda}\right)$ decreases from unity, becoming zero when $\frac{r}{\lambda} = 0.191$. Over this range of $\frac{r}{\lambda}$ the maxima of the y and I fringes coincide, but the contrast in I becomes zero when $\frac{r}{\lambda} = 0.191$. Since J_0 is negative for values of $\frac{r}{\lambda}$ in the range 0.191 to 0.438, a maximum of I occurs for values of X giving a minimum of y. The contrast at first increases with $\frac{r}{\lambda}$, attaining a maximum at $\frac{r}{\lambda} = 0.312$, and then recedes to zero. For values of $\frac{r}{\lambda}$ in the interval

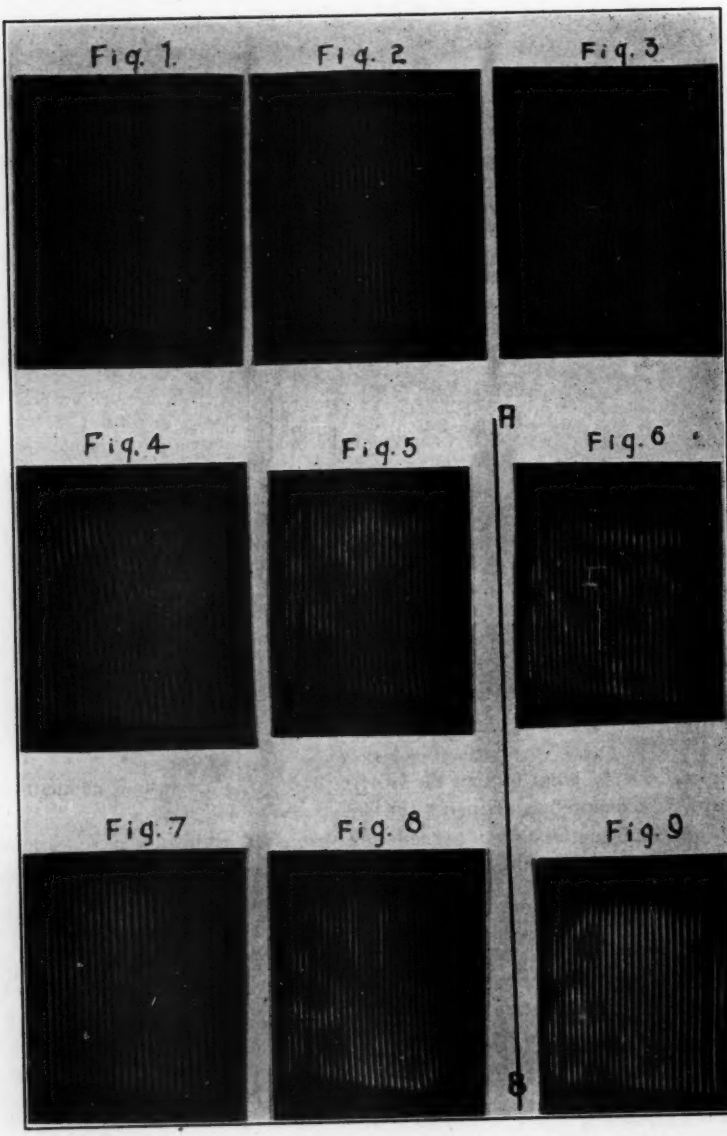


Fig. 1.
Fig. 2
Fig. 3
Fig. 4
Fig. 5
A
Fig. 6
Fig. 7
Fig. 8
Fig. 9
B

0.438 to 0.687 the maxima of I and y again coincide. The contrast is zero at each end of the interval, having as before a maximum somewhere in the interval. This behavior is repeated as $\frac{r}{\lambda}$ increases.

Consider the fringes appearing on a vibrating portion of the crystal surface. As the crystal is driven with greater amplitude, the fringes become less distinct until they are completely destroyed. At this point, the amplitude of vibration of the surface element is 0.191λ. With further increase in amplitude, fringes shifted by an amount $\frac{h}{2}$ first appear, rise to maximum contrast, and disappear as the amplitude of oscillation approaches the value $0.438\ \lambda$. This alternate fringe shift and repeated destruction of fringes occurs at definite intervals in $\frac{r}{\lambda}$, permitting not only an observation of the portions of the surface which are in motion, but also an estimate of the amplitude of crystal vibration to almost one-tenth of the wave-length of light employed.

Apparatus and experimental technique will be discussed in a later article, after more crystals have been examined. The accompanying illustrations have been taken from a crystal of Curie cut, the electric field parallel to the electric axis. In these illustrations, which represent but a few of the frequencies at which the crystal responds, the optic axis is in the plane of the paper and at right angles to the fringes.

The response of a pattern to sharper tuning, that is, to gradually increasing amplitude of oscillation, is illustrated by figures 1–4. In figure 4 the central portion of the lower of the two prominent oblong oscillating areas in 3 has reached such an amplitude of oscillation that the fringes on this portion are shifted by an amount $\frac{h}{2}$ with respect to the adjacent fringes. These are in turn shifted by $\frac{h}{2}$ with respect to the y fringes. The amplitude of oscillation of the central portion mentioned above is in the lower half of the range 0.438λ to 0.687λ where λ is 5461 Angströms.

The pattern 1–4 and that of figure 7 are of a type in which the crystal reacts vigorously to the applied voltage. In this type there is no apparent geometrical design and the pattern formed on the reverse surface at any particular frequency does not bear a strong resemblance to that formed on the front surface. On the other hand, figures 5 and 6 represent the front and back respectively at one frequency and 8 and 9 represent the front and back at another frequency, and in these cases a simple geometrical pattern is quite evident and the response of the front and back surfaces of the crystal is essentially the same under the same conditions of excitation. In these figures corresponding points of the front and back surfaces are

those situated symmetrically with respect to the line AB. The frequencies calculated from the spacing of the rows of spots on figures 5 and 6 using the velocity of sound do not agree with those used to drive the crystal, nor do they appear to be harmonics of the fundamental frequencies of the crystal as calculated in the usual way. It is hoped by later work to clear up this situation.

The crystal is found to respond but feebly at its fundamental. But the surface does not, as frequently supposed, oscillate as a whole.

Curves A and B (figure 10) show the relation between maximum amplitudes of oscillation and voltage (R. M. S.) applied across the crystal for two of the outstanding spots in the pattern shown in illustrations 1–4.

The writer wishes to express his indebtedness to Drs. C. E. Mendenhall, J. R. Roebuck and H. W. March for their interest and suggestions in connection with this work.

PERTURBATIONS IN BAND SPECTRA. II.

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In a previous article[1] on this subject (I), it was shown that certain perturbations in the CN bands can be explained as due to the close approach of terms with the same rotational quantum number, j, in two different electronic states. The unusual occurrence of a perturbation in the *final* ^2S state of the violet CN system led to the discovery of a corresponding perturbation in the initial state (^2P) of the red CN system. These results were in good agreement with the theoretical deductions of Kronig.[2] At about the same time Dieke[3] found a similar verification of this theory in the He_2 bands, where it was possible to fix accurately the relative positions of the two sets of mutually perturbing levels. The present work deals with a number of perturbations in the CO Ångström bands, $^1S \longrightarrow {}^1P$, not hitherto described. The rotational structure of this system was first analyzed in terms of the quantum theory by Hulthén.[4] He applied the combination principle in an approximate form (Equation (2), below) to the (0,1) (0,2) (0,3) and (1,1) bands, but was unable to get any analysis of the bands (0,0) and (1,0). The latter evidently have an abnormal structure. Jassé[5] gave a partial analysis of them, showing that for low quantum numbers the regular P, Q and R branches are present. Two

additional short series (3 members in each) were also reported as forming part of the structure of these two bands. In the recent work of Johnson and Asundi[6] the extra lines are regarded as of extraneous origin, but only an incomplete analysis of these bands could be given. These authors were also able to establish definitely that the origins of the (0,1) and (1,1) bands lie about 4.2 cm.$^{-1}$ toward lower frequencies from their positions calculated by a smooth formula representing the origins of the other bands. This was referred to as a "vibrational perturbation" of the state $n'' = 1$. Birge[7] had previously suspected that the $n'' = 0$ level is thus displaced. From our work it now appears that all of these irregularities are due to perturbations of the rotational terms of the ^1P state, of a type similar to those we have previously reported.

In the quantum analysis of the Ångström bands given by Hulthén, assignments of the lines extended usually to about $j = 25$. We have examined the band structure at higher rotational quantum numbers, and find that all bands with $n'' = 0$, 1, 2 and 3 show marked perturbations in the region $j = 25$ to 40, and some at even lower j's. A set of grating plates taken originally for a study of the Zeeman effect[8] was kindly furnished us for this work by Dr. F. H. Crawford. To investigate these perturbations by the method of plotting deviation curves which was used before, new measurements were made of the band lines in parts of the (0,1), (0,2) and (0,3) bands, starting usually about 10 lines before the first perturbation, and extending as far as the lines could be traced. It was found necessary to re-measure all the lines of the (0,0) band. Formulae were derived, usually by the method of least squares, to fit the regular lines of the band, and the deviations of all lines, regular and perturbed, from these formulae were represented graphically. The form of the curves at the perturbations (see figure 1, below) was always of the characteristic "resonance" type found in our previous work. In approaching a perturbation from the side of low j's, the lines invariably deviate first toward higher frequencies. If, as is usual, these perturbations were in the initial state, ^1S, the deviation curves should be identical for $R(j-1)$, $Q(j)$ and $P(j+1)$. This is not the case. They are always identical for $R(j)$ and $P(j)$, while the Q branch is perturbed at a *different* j value. Hence the perturbations are in the lower state, ^1P. According to Kronig's theory this is possible here, because the ^1P state is an excited state, and hence, even for the lowest vibrational states, coincidences may occur with the levels of equal j in some other electronic state. That the Q branch should not be perturbed at the same value of j'' as the P and R branches also follows from the theory when we recall that the rotational levels of a ^1P state are doublets (σ-type doubling), one of the components of each being an odd, and the other an even state. The same phenomenon was observed in the perturbation of the ^2P levels of CN described in I. Space will not permit us to present here the results

on all the perturbations, but only those bearing on the anomalous features mentioned above. The details will be reserved for a more complete publication elsewhere. It may be of value, however, to indicate the values of j at which the perturbations thus far established reach their maximum displacements:

n''	P,R branches	Q branch
0	j = 9, 16, 27 and >30	12, 29 and >35
1	(0), 26 and >35	(0), 24 and >29
2	25, 33	29
3	31	28, 34

Considering first the (0,0) band, we give in figure 1 (a) (continuous curves) the deviations from the equation

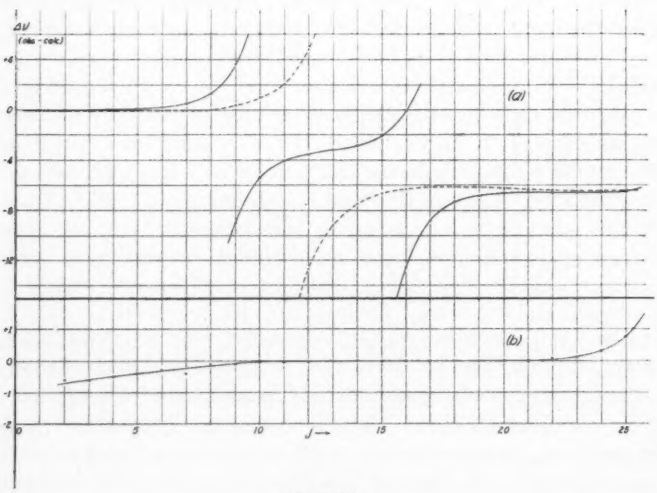

FIGURE 1

$$R(j) = 22{,}175.315 + 4.1943j + 0.36766j^2 - 0.000546j^3 \qquad (1)$$

of the lines assigned to the R branch. If from the R lines thus chosen, the corresponding P lines are computed from $P(j) = R(j) - \Delta_2F'(j)$, the last term being known from other bands, a line of the proper intensity always appears in the calculated position. This must be the case for a correct assignment of the lines, since this relation is exact, and not affected by perturbations or σ-type doubling. In the analyses of Jassé,[5] and of Johnson and Asundi,[6] no attention was apparently paid to this requirement, but the lines were chosen to satisfy the approximate relation

$$R(j) - Q(j) \cong Q(j+1) - P(j+1), \qquad (2)$$

with an attempt to keep the combination defect (σ-type doubling) within reasonable limits. That this leads to false results is apparent in table

TABLE 1

NEW ANALYSIS OF THE (0, 0) BAND

	$P(j)$		$Q(j)$		$R(j)$	
j^*	a	b	a	b	a	b
1	22,168.10		22,172.10			
2	165.69		173.44			
3	163.89		175.57		22,191.30	
4	162.90		178.38		197.94	
5	162.63		181.99		205.41	
6	163.17		186.28		213.79	
7	164.60		191.30		222.99	
8	167.25		197.10		233.45	
9	172.10	159.48	203.84		246.09	233.45
10		166.32	211.55			248.11
11		171.65	220.63	200.73		261.16
12		176.87	231.85	214.38		274.08
13		182.54	246.09	227.05		287.59
14		188.97	263.48	238.77		301.79
15		196.47		250.26		317.02
16	193.44	205.92		261.92	321.80	334.21
17	205.41			274.08	341.39	
18	215.59			286.77	359.43	
19	225.59			300.08	377.21	
20	235.97			314.08	395.28	
21	246.94			328.73	413.69	
22	258.45			344.02	433.16	
23	270.62			360.05	453.07	
24	283.56			376.76	473.64	
25	297.22			394.21	494.99	

TABLE 2

$\Delta_2F'(j)$ FROM (0,0) BAND

Δ_2F' (OBS.)			(OBS., JOHNSON AND ASUNDI)	(CALC.*)		(OBS.)		
j	a	b			j	a	b	(CALC.*)
1			11.62	11.68	14		112.81	112.80
2			19.50	19.47	15		120.55	120.55
3	27.41		27.28	27.26	16	128.36	128.29	128.30
4	35.04		35.09	35.05	17	135.98		136.05
5	42.78		43.20	42.83	18	143.84		143.79
6	50.62		50.96	50.62	19	151.62		151.53
7	58.39		58.29	58.40	20	159.31		159.26
8	66.20		67.03	66.18	21	166.75		166.98
9	73.99	73.97	75.13	73.96	22	174.71		174.70
10		81.79	...	81.74	23	182.45		182.41
11		89.51		89.51	24	190.08		190.11
12		97.21		97.27	25	197.77		197.81
13		105.05		105.04				

* Calculated values from $\Delta_2F'(j) = 4B'(j + 1/2) + 8D'(j + 1/2)^3$, $B' = 1.9476$; $D' = -6.444 \times 10^{-6}$.

II, below, from the failure of the combination relation for the lines of higher j assigned by Johnson and Asundi. Since the relation is fulfilled for the lines chosen here, the R and P branches have the same curve of deviations, as of course they must. Calculated values for the Q lines were next obtained from $Q(j) \cong R(j) - \Delta_1 F'(j)$, where $R(j)$ is the *calculated* value from equation (1). Lines were found approximately in these positions, and their deviations are represented by the broken curves of figure 1(a). The complete assignment is given in table 1, and supported by the combination differences of table 2. In certain cases two lines are recorded (columns a and b) for the same j value. This will be briefly discussed below.

Evidently when perturbations of this magnitude are present, the analysis of the band structure which is most probably correct is that which gives smooth deviation curves of the type shown in figure 1, and which at the same time satisfies the exact combination relations. The failure of Jassé, and of Johnson and Asundi, to identify the proper P and R lines above $j = 8$, and Q lines above $j = 10$, results from the enormous increase in the combination defect (difference between the continuous and broken curves at a given value of j) in the region of a perturbation. According to our analysis, which leaves no lines in the region of low j's unaccounted for, the two extra branches found by Jassé are spurious. For the (0,0) band, the proper assignment of these lines is as follows:

m	$P'(m)$ (Jassé)	New assignment	$R'(m)$ (Jassé)	New
1	...	...	22,177.0	$P(12)$
2	22,167.4	$P(8)$	182.7	$P(13)$
3	162.8	$P(5)$	189.1	$P(14)$
4	159.7	$P(9)b$	...	...

The perturbations in the (0,1) band are also of interest, since in the final state, $n'' = 1$, the rotational levels are supposed to be displaced as a whole by about 4.2 cm^{-1}. This follows from the fact that the frequencies of the head and of the origin of this band are found[6] to be lower by this amount than is to be expected from formulae derived to fit the other bands of the system. In figure 1(b) are given the deviations of the R lines from the equation $R(j) = 20{,}687.18 + 4.58j + 0.360\ j.^2$ The slight negative trend at the lowest values of j is significant. The curve here evidently has the same form as that for the R lines of the (0,0) band between, let us say, $j = 18$ and 25. If only lines above $j = 18$ of the latter band could be observed, one would conclude that its origin is perturbed by about 6.5 cm^{-1} toward lower frequencies. The fact that this is of the same order of magnitude and in the same sense as the "vibrational" perturbation of the state $n'' = 1$ indicates that the latter is due to a similar perturbing influence, though shifted to lower j values. On this view the vibrational perturbation is to be correlated with the *constant* displacement of the

lines which is often observed (see also figure 1 of I for an analogous effect in CN) after they have passed through the discontinuity and have become more or less regular again. This feature of the perturbation curves has as yet reached no theoretical interpretation, but it seems doubtful whether such displacements can be attributed solely to a perturbation of the vibrational energy, as distinguished from the rotational energy.

All of the phenomena considered above indicate, according to Kronig's theory,[2] that the rotational terms of the ^1P state are crossed by terms of another electronic state, in such a way that terms of equal j come into close coincidence. The electronic states of CO are known very completely, and a study of the various possibilities shows that it is the a state (final state of the 3rd positive carbon bands and initial state of the Cameron bands) which is involved here. The a' state, which lies just below the $^1P(A)$ state, is ruled out by the following considerations. First, the vibrational levels of the a' state, as fixed approximately by the heads of the ultra-violet absorption bands, are so situated that none coincide closely with the low-n levels of ^1P. In order for terms of the same j to coincide, it is obvious that the vibration levels to which these terms belong must be fairly close together. Second, the vibration frequency in a' is very different from that in ^1P (1155 as compared to 1499). In view of the general parallelism which is known to exist between the vibration frequency and the moment of inertia, this means that the latter is also quite different in the two states. When the perturbations are as large as those observed here, one infers that the moment of inertia, and hence the value of B, must be very nearly the same as in the perturbing state, for only then will terms with the same j approach closely over a considerable range of j values. On the other hand, neither of the above objections applies to the a state, which is next below a'. A rough extrapolation of its vibrational levels shows that there is probably a close approach of the $n = 10$ level to that of $n = 0$ for ^1P. Furthermore the extrapolated vibration frequency, $\omega_{10^1/_2}$, is 1420, as compared to 1481.8 for $\omega_{1/_2}$ for the ^1P state; the values are thus close enough to account for large perturbations. On this view, it seems probable that the perturbation at $j = 12$ in the (0,0) band and that at low j's in the (0,1) band may be ascribed to the successive vibration levels $n = 10$, 11 of the a state, since the shift to a lower j value is consistent with the slightly smaller vibration frequency in the a state.

An important theoretical objection arises to the conclusion that the a levels are responsible for the perturbations. One of the requirements for two terms to perturb each other is that they must belong to electronic states of the *same multiplicity*.[2] The a state has usually been designated ^3P,[9] although, according to recent work by Johnson,[10] it is ^5P. There is no doubt, however, that it is not a singlet state. Hence our findings are not in agreement with the theory on this point, and it seems hardly possible

to interpret them otherwise. Besides the evidence mentioned above, there is the fact that these perturbations are multiple, occurring at several values of j close together. The case shown in figure 1(a) is typical. The perturbation in the P and R branches is evidently double ($j = 9, 16$), and in the Q branch single ($j = 12$). The same effect is found in the (0,2) band—$P, R : j = 25, 33$; $Q : j = 29$—and possibly in still other cases, although there are certain complications. It is not clear how this is to be explained by assuming a singlet state as the perturbing one.

In conclusion, it may be well to mention a suggestion made to the writers by Prof. E. C. Kemble with regard to the occurrence of two lines with the same j near the maximum of a perturbation. An example is in the Q branch of the (0,0) band between $j = 11$ and 14. In any perturbation we are dealing with two sets of terms which, when the energy is plotted against j, would cross at a certain value of j, but actually are displaced here due to a "perturbation energy," as shown by Kronig. From the standpoint of the wave mechanics the two electronic states lose their identity in this region, and an interpretation of the various quantum numbers in terms of the molecular model is no longer possible. As a result, transitions from *both* sets of levels begin to take place, and extra lines may occur in the region of the perturbation where the displacements are largest. Our observations show that the intensity of the lines decreases as the displacements increase, and that the sum of the intensity of the two transitions for a given j is about equal to that which would be expected for an unperturbed line. In further work, a quantitative study will be made of these intensity relations, as well as of the Zeeman effect of the perturbed terms.

[1] J. E. Rosenthal and F. A. Jenkins, these PROCEEDINGS, **15**, 381 (1929). *Cf.* also *Phys. Rev.*, **33**, 285 (1929) (abstract).

[2] R. de L. Kronig, *Zeits. Physik*, **50**, 347 (1928).

[3] G. H. Dieke, *Nature*, **123**, 446 (1929).

[4] E. Hulthén, *Ann. Physik*, **71**, 41 (1923).

[5] Mlle. Jassé, *Comptes rend.*, **182**, 692 (1926).

[6] R. C. Johnson and R. K. Asundi, *Proc. Roy. Soc.*, **123A**, 560 (1929).

[7] R. T. Birge, *Phys. Rev.*, **28**, 1169 (1926).

[8] E. C. Kemble, R. S. Mulliken and F. H. Crawford, *Phys. Rev.*, **30**, 438 (1928). F. H. Crawford, *Phys. Rev.*, **33**, 341 (1929).

[9] R. T. Birge, Ref. 7, p. 1160.

[10] R. C. Johnson, *Transactions Faraday Soc.*, in press.

ON GROUPS DEFINED BY $A^q = 1, B^{-1}AB = A^x, B^Q = A^e$

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In this note we shall investigate some of the properties of the solvable, non-abelian groups generated as follows:

$$A^q = 1, B^{-1}AB = A^x, B^Q = A^e, \tag{1}$$

where B^Q is the least power of B found in the group $\{A\}$. Evidence that these groups include a large number and a great variety of groups is given by the fact that more than one half of the solvable, abstract groups of order less than 64 may be generated in this manner. Hence simple or complete theorems are scarcely to be expected. A normalization, found by Professor Dickson, of the generators of these groups and some necessary conditions for the simple isomorphism of these groups will be given.

Professor Dickson's normalization of the generators of these groups may be stated as follows:

The generators of the group defined by (1) may be so chosen that the exponent e is a divisor of q.

This normalization will now be established. First note that unless e and q have a H.C.F. >1, A is a power of B and the group is cyclic and thus not of the given type. Let $e = \alpha\beta$ and $q = \alpha\gamma$, where $(\beta,\gamma) = 1$. Then

$$B^{Qm} = A^{em} = A^{\alpha},$$

if

$$\beta m \equiv 1, \text{ mod } \gamma. \tag{2}$$

Since β is prime γ, there exists a solution $m_1 = m$ of (2). If this solution, prime to γ, is also prime to Q, choose it for m. If it is not, note that all solutions of (2) are of the form $m' = m_1 + i\gamma$, $i = 0, 1, 2, \ldots$. By Dirichlet's Theorem, there are an infinite number of primes $\equiv m_1$, mod γ, and hence there exists at least one solution m' prime to both Q and γ. If now $B^{m'}$ is chosen as a new B, $x^{m'}$ as a new x, and A is retained, the group may be written in the desired form.

The problem of determining when two groups G, and G', of the type under discussion are simply isomorphic is a difficult one. Some necessary conditions for the simple isomorphism of two such groups will now be determined. Let G be defined by (1) and G' by

$$T^{q'} = 1, S^{-1}TS = T^{x'}, S^{Q'} = T^{e'}. \tag{3}$$

Suppose the two groups simply isomorphic. Then in the isomorphism

let the operation T correspond to the operation B^sA^r, the operation S to B^uA^t, the operation A to S^gT^h, and the operation B to S^kT^l. The symbol $\sim$ between two operations will henceforth be used to denote that these operations correspond to each other in the simple isomorphism between G and G'.

First it will be shown that if w is the least integer such that

$$w(x - 1) \equiv 0, \text{ mod } q. \tag{4}$$

it is equal to the least integer w' such that

$$w'(x' - 1) \equiv 0, \text{ mod } q'. \tag{4'}$$

By (1), $B^{-s}A^mB^s = A^{mx^s}$. Hence (3_2) gives

$$T^{x'} \sim B^sA^{rx^u+t(1-x^s)},$$

and hence

$$T^{x'-1} \sim A^{r(x^u-1)+t(1-x^s)}. \tag{5}$$

From (4) and (5),

$$w(x' - 1) \equiv 0, \text{ mod } q',$$

and hence w is a multiple of w' by (4'). On the other hand, since

$$A^{x-1} \sim T^{h(x'^k-1)+l(1-x'^g)} \tag{6}$$

$$w'(x - 1) \equiv 0, \text{ mod } q,$$

and w' is a multiple of w. Therefore $w = w'$.

It will also be shown that if x belongs to the exponent E, modulo q, then x' belongs to the exponent E, modulo q'. The proof of this property depends upon some auxiliary congruences which will now be established. Since the operation B^sA^r is transformed into a power of itself by every operation in G, and thus in particular by A, while $A^{-1}B^sA^rA = B^sA^rA^{1-x^s}$ A^{1-x^s} is a power of the former operation. Likewise it has been shown in (6) that A^{x-1} is in the group $\{B^sA^r\}$. Thus since

$$A^{1-x}B^sA^rA^{x-1} = B^sA^{r+(x-1)(1-x^s)},$$

the congruence

$$(x - 1)(x^s - 1) \equiv 0, \text{ mod } q \tag{7}$$

holds. Thus

$$(x^s - 1)^2 \equiv 0, \text{ mod } q,$$

and hence

$$x^{ms} - 1 \equiv m(x^s - 1), \text{ mod } q. \tag{8}$$

Now

$$S^{-m}TS^m = T^{x'^m} \sim B^s A^{rx^{um}+t(1-x^s)(x^{(m+1)u}+\ldots+x^u+1)}.$$

By (7) and (8)

$$B^s A^{rx^{um}+t(1-x^s)(x^{(m-1)u}+\ldots x^u+1)} = B^s A^{rx^{um}+t(1-x^{sm})}.$$

Hence

$$T^{x'^m-1} \sim A^{r(x^{um}-1)+t(1-x^{sm})}. \tag{9}$$

Now let E be the exponent to which x belongs, modulo q, and E' be the exponent to which x' belongs, modulo q'. From (9), it is seen that

$$x'^E - 1 \equiv 0, \text{ mod } q'.$$

Thus E is a multiple of E'. Likewise it is easily seen that

$$A^{x^m-1} \sim T^{h(x'^{km}-1)+l(1-x'^{lm})}$$

and hence E' is a multiple of E. Thus $E' = E$.

It should be noted that B^E is the least power of B commutative with A and that A^W is the least power of A commutative with B. Hence the central of G is $\{B^E, A^W\}$, and it is seen that the central of G' is $\{S^E, T^W\}$.

The above discussion may be summarized as follows:

The two groups,

$$G : A^q = 1, B^{-1}AB = A^x, B^Q = A^e$$

and

$$G' : T^{q'} = 1, S^{-1}TS = T^{x'}, S^{Q'} = T^{e'},$$

of order $qQ = q'Q'$, are simply isomorphic only if

1. *The least integer w such that*

$$w(x - 1) \equiv 0, \text{ mod } q$$

is equal to the least integer w' such that

$$w'(x' - 1) \equiv 0, \text{ mod } q';$$

2. *x and x' both belong to the same exponent E, modulo q and q', respectively;*

3. *The central of $G = \{B^E, A^W\}$ is simply isomorphic to the central of $G' = \{S^E, T^W\}$.*

Further conditions on the integers r, s, t, u, defined above, have been found, but while useful in determining the existence of a simple isomorphism in specific cases, they do not lead to any general theory.

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ON THE EXISTENCE OF INTEGRALS OF EINSTEIN'S GRAVITATIONAL EQUATIONS FOR FREE SPACE AND THEIR EXTENSION TO n VARIABLES

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§ 1. The well-known form of Einstein's gravitational equations for free space, namely,

$$R_{\alpha\beta} = 0, \qquad \text{(a)}$$

expresses the fact that the contracted curvature tensor $R_{\alpha\beta}$ (or $B^{i}_{\alpha\beta i}$) is equal to zero. Written at greater length, equations (a) become

$$\frac{\partial \Gamma^{i}_{\alpha\beta}}{\partial x^{i}} - \frac{\partial \Gamma^{i}_{\alpha i}}{\partial x^{\beta}} + \Gamma^{\sigma}_{\alpha\beta}\,\Gamma^{i}_{\sigma i} - \Gamma^{\sigma}_{\alpha i}\,\Gamma^{i}_{\sigma\beta} = 0,$$

where the Γ's are Christoffel symbols with respect to the coefficients $g_{\alpha\beta}(x)$ of a quadratic differential form. Hence (a) is a system of partial differential equations of the second order in a set of unknown functions $g_{\alpha\beta}$. Investigations of the solutions of this system of equations have been confined, so far as I am aware, to solutions possessing certain symmetrical properties. The most important of these special solutions is that given by Schwarzschild,[1] which postulates spherical symmetry about a point in the four-dimensional world and which is used in the calculation of the rotation of the perihelion of Mercury; there are also the axial symmetric solutions investigated by Weyl and others.[2] It is the object of this note to consider the existence of the general integral $g_{\alpha\beta}$ $(= g_{\beta\alpha})$ of the system (a) in $n(\geqq 2)$ variables $x^{1}, \ldots, x^{n}$ so as to include in particular the case $n = 4$ occurring in the theory of relativity. In order to eliminate the arbitrariness in the functions $g_{\alpha\beta}$ due to the fact that the equations (a) are invariant under arbitrary transformations of coördinates, we select a particular system of coördinates, namely, a system of normal coördinates $y^{1}, \ldots, y^{n}$, and require that the $g_{\alpha\beta}$ have the values $\pm \delta^{\alpha}_{\beta}$ at the origin of this system. The general integral $g_{\alpha\beta}$ of the system of equations (a) will then depend on a number of arbitrary functions which are specified precisely in the existence theorem in §6. A possible extension of this theorem to the case where equations (a) are considered with reference to arbitrary coördinates x^{i}, is indicated in my note on a related investigation on page 850 of this volume of these PROCEEDINGS.

§ 2. Consider the Christoffel symbols

$$\Gamma^{i}_{\alpha\beta} = \frac{1}{2}\, g^{i\sigma}\left(\frac{\partial g_{\alpha\sigma}}{\partial y^{\beta}} + \frac{\partial g_{\sigma\beta}}{\partial y^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial y^{\sigma}}\right),$$

taken with respect to a system of normal coördinates y^i. If we differentiate both members of these equations and evaluate at the origin, we obtain

$$A^i_{\alpha\beta\gamma} = \frac{1}{2} g^{i\sigma} (g_{\alpha\sigma,\beta\gamma} + g_{\sigma\beta,\alpha\gamma} - g_{\alpha\beta,\sigma\gamma}),$$

where the $A^i_{\alpha\beta\gamma}$ are the components of the normal tensor and the $g_{\alpha\beta,\gamma\delta}$ are the components of the second extension of the fundamental tensor $g_{\alpha\beta}$. We now deduce from the identities

$$B^i_{\alpha\beta\gamma} = A^i_{\alpha\beta\gamma} - A^i_{\alpha\gamma\beta},$$

where the $B^i_{\alpha\beta\gamma}$ are the components of the curvature tensor and the identities (c) in §3 that

$$R_{\alpha\beta} = -\frac{3}{2} g^{\gamma\delta} g_{\alpha\beta,\gamma\delta}.$$

Hence the system of equations (a) and the system

$$g^{\gamma\delta} g_{\alpha\beta,\gamma\delta} = 0 \qquad (b)$$

are completely equivalent. In the following investigation we shall replace the system of equations (a) by the system (b) which possesses certain practical advantages over system (a) and which will enable us to make direct use of the results of an earlier article.[3]

§ 3. A set of components $g_{\alpha\beta,\gamma\delta}$ will be said to be independent if they can be taken as arbitrary after the algebraic conditions[4]

$$g_{\alpha\beta,\gamma\delta} = g_{\beta\alpha,\gamma\delta} = g_{\alpha\beta,\delta\gamma};\ g_{\alpha\beta,\gamma\delta} + g_{\alpha\gamma,\delta\beta} + g_{\alpha\delta,\beta\gamma} = 0 \qquad (c)$$

have been imposed. It is possible to divide the independent components $g_{\alpha\beta,\gamma\delta}$ into mutually distinct groups G_m by the following

Rule.[3] *The group* G_m $(m = 0, 1, \ldots, n-2)$ *for the components* $g_{\alpha\beta,\gamma\delta}$ *is composed of all components that can be formed from* $g_{\alpha\beta,\gamma\delta}$ *by taking* $\alpha = m + 1$; $\beta,\gamma,\delta = 1, \ldots, n$ *subject to the inequalities* $\beta \leqq m + 1$, $\beta < \gamma$, $\gamma \leqq \delta$ *and* $\delta > m + 1$. If $K(p, q)$ is used to denote the number of combinations with repetitions of p things taken q at a time, then the number B_m of components $g_{\alpha\beta,\gamma\delta}$ in group G_m is $(m + 1)\,K(n, 2) - nK\,(m + 1, 2)$. The designation B_{lm} $(l = 1, \ldots, B_m)$ will be used for the components $g_{\alpha\beta,\gamma\delta}$ in group G_m.

The components of the covariant derivative $g_{\alpha\beta,\gamma\delta,\epsilon}$ satisfy the following set of identities:

$$2g_{\alpha\beta,\gamma\delta,\epsilon} = g_{\alpha\beta,\gamma\delta,\alpha} + g_{\alpha\epsilon,\gamma\delta,\beta} + g_{\alpha\beta,\epsilon\delta,\gamma} + g_{\alpha\beta,\gamma\epsilon,\delta}. \qquad (d)$$

On the basis of these identities it can be shown that a system of equations of the form

$$\frac{\partial B_{lm}}{\partial y^k} = \sum \frac{\partial B_{pq}}{\partial y^r} + \bigstar \qquad \text{(e)}$$

$$\begin{pmatrix} m = 1, \ldots, n-2 \\ l = 1, \ldots, B_m \\ k = 1, \ldots, m \end{pmatrix}$$

must be satisfied, where $\sum$ represents a linear expression in the derivatives $\partial B_{pq}/\partial y^r$ and the $\bigstar$ terms denote polynomials in the B_{ij} and the components of affine connection; the derivatives $\partial B_{pq}/\partial y^r$ in the right member of any equation (e) are such that the inequalities $q \leqq m$, $r > q$ are satisfied.[3]

Lemma. In any equation (e) *the derivatives of the components* $g_{\alpha\beta,\gamma\delta}$ *corresponding to* $\partial B_{pq}/\partial y^r$ *result from rearrangements of the indices* $\alpha\beta\gamma\delta\epsilon$ *in the derivative* $\partial g_{\alpha\beta,\gamma\delta}/\partial y^\epsilon$ *which corresponds to the left member of the equation.* This fact which is obvious from the method of derivation of equations (e) will be used later.

Due to the fact that the components $g_{\alpha\beta,\gamma\delta}$ are symmetric in the indices α,β there are $K(n, 2)$ independent equations (b). Those equations (b) can be taken as independent for which $\alpha \leqq \beta$ and for definiteness this condition will be assumed to be satisfied.

§ 4. If $n = 2$ there is only one independent component $g_{11,22}$ which belongs to group G_0 and this must necessarily vanish as a consequence of (b). For $n = 3$ the independent components g comprise three components $g_{11,22}$; $g_{11,23}$; $g_{11,33}$ belonging to group G_0 and three components $g_{21,22}$; $g_{21,33}$; $g_{22,33}$ belonging to group G_1. These six components must vanish since, for $n = 3$, there are six independent equations in the system (b). If $n = 2$ or 3 the space with line element defined by the quadratic differential form $g_{\alpha\beta}dx^\alpha d_x\beta$, the coefficients of which satisfy the system of equations (b), is therefore flat.

§ 5. In this and the following sections the assumption $n \geqq 4$ will be made. By covariant differentiation of (b) we obtain

$$g^{\gamma\delta}g_{\alpha\beta,\gamma\delta,\epsilon} = 0. \qquad \text{(f)}$$

Under the above restriction $\alpha \leqq \beta$ there are $nK(n, 2)$ equations in the system (f); not all of these equations, however, are independent. In fact, if we multiply (d) through by $g^{\alpha\beta}$, $g^{\gamma\delta}$ and sum on the indices $\alpha\beta\gamma\delta$ we find

$$2\, g^{\alpha\beta}\, X_{\epsilon\alpha\beta} = g^{\alpha\beta}\, X_{\alpha\beta\epsilon},$$

where $X_{\alpha\beta\epsilon}$ has been used to denote the left member of (f). A consideration of these latter identities shows that the n quantities $X_{1\beta 1}$ are linearly dependent on the remaining quantities X provided that $g^{11} \neq 0$, as will be assumed. Hence the n equations

$$X_{i\beta i} = 0 (\beta = 1, \ldots, n)$$

can be omitted from the system (f).

§ 6. If we expand the covariant derivatives in (f) by the formula for covariant differentiation these equations can be written

$$g^{\gamma\delta} g_{\alpha\beta,\gamma\delta/\epsilon} = \bigstar \qquad (\alpha \neq 1 \text{ if } \epsilon = 1) \qquad (g)$$

where $g_{\alpha\beta,\gamma\delta/\epsilon}$ has been used to denote the ordinary partial derivatives of the component $g_{\alpha\beta,\gamma\delta}$ with respect to y^ϵ. Let us now assume that for $y^i = 0$ the component $g_{\alpha\beta}$ has the value zero if $\alpha \neq \beta$ and the value $+1$ or -1 if $\alpha = \beta$ (this is in agreement with the assumption in § 5 that the condition $g^{11} \neq 0$ be satisfied). We wish to show that for $y^i = 0$ the systems (e) and (g) can be solved for the following derivatives of the components $g_{\alpha\beta,\gamma\delta}$: the derivatives $g_{11,\alpha\beta/\epsilon}$ for $\alpha,\beta = 2, \ldots, n$; $\epsilon = 1, \ldots, n$ which comprise all derivatives of components $g_{\alpha\beta,\gamma\delta}$ belonging to group G_0; the derivatives $g_{22,33/\epsilon}$; $g_{21,33/\epsilon}$; $g_{21,2\beta/\epsilon}$ for $\beta = 3, \ldots, n$; $\epsilon = 2, \ldots, n$ comprising certain derivatives of components $g_{\alpha\beta,\gamma\delta}$ in group G_1; and finally the derivatives in the left members of equations (e) themselves. For this purpose we divide the system (g) into the following three systems of equations which we consider at the origin of coördinates $y^i = 0$. First the system

$$g_{11,\alpha\beta/\epsilon} \pm g_{22,\alpha\beta/\epsilon} \pm \ldots \pm g_{nn,\alpha\beta/\epsilon} = \bigstar; \qquad (I)$$
$$\begin{pmatrix} \alpha,\beta = 2, \ldots, n \\ \epsilon = 1, \ldots, n \end{pmatrix}$$

second the system

$$\begin{aligned} g_{12,33/\epsilon} \pm g_{12,44/\epsilon} \pm \ldots \pm g_{12,nn/\epsilon} &= \bigstar, \\ g_{13,22/\epsilon} \pm g_{13,44/\epsilon} \pm \ldots \pm g_{13,nn/\epsilon} &= \bigstar, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \qquad (II) \\ g_{1n,22/\epsilon} \pm g_{1n,33/\epsilon} \pm \ldots \pm g_{1n,n-1n-1/\epsilon} &= \bigstar; \\ (\epsilon = 2, \ldots, n) & \end{aligned}$$

and finally the system

$$g_{11,22/\epsilon} \pm g_{11,33/\epsilon} \pm \ldots \pm g_{11,nn/\epsilon} = \bigstar. \qquad (III)$$
$$(\epsilon = 2, \ldots, n)$$

The derivatives in (III) occur also in certain equations of the system (I). On eliminating these derivatives from (III) by means of (I) we have a set of equations of the form

$$\begin{aligned} & g_{33,22/\epsilon} \pm \ldots \pm g_{nn,22/\epsilon} \\ & + g_{22,33/\epsilon} \pm \ldots \pm g_{nn,33/\epsilon} \\ & \ldots\ldots\ldots\ldots\ldots\ldots \\ & \ldots\ldots\ldots\ldots\ldots\ldots \\ & \pm g_{22,nn/\epsilon} \pm \ldots \pm g_{nn,n-1n-1/\epsilon} = \bigstar, \end{aligned}$$

which can be solved for the derivatives $g_{22,33/\epsilon}$ where $\epsilon = 2, \ldots, n$ and these latter equations can be used to eliminate the derivatives $g_{22,33/\epsilon}$ ($\epsilon = 2, \ldots, n$) from (I). When this has been done the equations of systems (I) and (II) can be solved for the derivatives $g_{11,\alpha\beta/\epsilon}$ ($\alpha,\beta = 2, \ldots, n$; $\epsilon = 1, \ldots, n$) and $g_{12,33/\epsilon}$; $g_{13,22/\epsilon}$; $\ldots$; $g_{1n,22/\epsilon}$ ($\epsilon = 2, \ldots, n$) by a simple transposition of the remaining terms. In this solution let us now replace the derivative $g_{1\beta,22/\epsilon}$ where $\beta = 3, \ldots, n$; $\epsilon = 2, \ldots, n$ by -2 times the derivative $g_{21,2\beta/\epsilon}$ in accordance with (c). Certain equations of the system which we have thus obtained by solving (g) for the above derivatives will contain in their right members derivatives occurring in the left members of (e). On eliminating these derivatives by the substitution (e) we obtain a system of equations which will be referred to as the system R_1. Let us now divide the system R_1 into two separate systems: (1) the system S_1 composed of those equations of R_1 which contain the derivatives $g_{22,33/\epsilon}$; $g_{21,33/\epsilon}$; $g_{21,23/\epsilon}$; $\ldots$; $g_{21,2n/\epsilon}$ where $\epsilon = 2, \ldots, n$ in their left members, and (2) the system T_1 composed of the remaining equations of R_1. By the aid of the lemma in § 3 it can be seen that none of the derivatives in the left members of R_1 can occur in the right members of any of the equations of the system S_1; also the derivatives $g_{11,\alpha\beta/\epsilon}$ ($\alpha, \beta = 2, \ldots, n$; $\epsilon = 1, \ldots, n$) do not occur in the right members of T_1. Hence the derivatives in the left members of S_1 can be eliminated from the right members of T_1 by a substitution involving the equations of the system S_1. Let us say that the system R_1 becomes the system R_2 as the result of this latter substitution. The equations of the system R_2 can now be used to eliminate the derivatives in the left members of R_2 from the right members of (e); the equations resulting from (e) in consequence of this elimination will be called the system R_3. The two systems R_2 and R_3 constitute the solution of (e) and (g) which we wished to obtain.

Now the fact that it is possible to solve equations (e) and (g) for the above derivatives when $g_{\alpha\beta}$ has the value $\pm \delta^\alpha_\beta$ shows that the determinant formed from the coefficients of these derivatives in (e) and (g) does not vanish identically as a function of $g_{\alpha\beta}$. Hence it is possible to solve (e) and (g) for the above derivatives for values of $g_{\alpha\beta}$ in the neighborhood of the values $\pm \delta^\alpha_\beta$. Let us now designate by B_{ln}, where $l = 1, \ldots, B_0 + n$, the individual components of the set $g_{22,33}$; $g_{21,33}$; $g_{21,23}$; $\ldots$; $g_{21,2n}$; and $g_{11,\alpha\beta}$ ($\alpha,\beta = 2, \ldots, n$); the $B_1 - n$ remaining components in group G_1 will be denoted by B_{l1} ($l = 1, \ldots, B_1 - n$) and the designation B_{lm} ($l = 1, \ldots, B_m$) for the components $g_{\alpha\beta,\gamma\delta}$ in group G_m ($m = 2, \ldots, n-2$) will remain unchanged. Then in virtue of the above solution of (e) and (g) we can write

$$\frac{\partial B_{lm}}{\partial y^k} = \sum \frac{\partial B_{pq}}{\partial y^r} + \bigstar \qquad \text{(h)}$$

$$\left[\begin{array}{c} m = 1, \ldots, n-2; \text{ or } m = n \\ l = 1, \ldots, C_m; \; k = 1, \ldots, m \\ \text{where} \\ C_1 = B_1 - n; \; C_n = B_0 + n \\ C_m = B_m \text{ otherwise,} \end{array}\right]$$

in which the summation $\sum$ in any equation denotes a linear expression in the derivatives $\partial B_{pq}/\partial y^r$ with coefficients which are rational functions of the components $g_{\alpha\beta}$. As in the preceding equations the ★ terms in (h) denote expressions in the $g_{\alpha\beta}$, their first derivatives, and the components $g_{\alpha\beta,\gamma\delta}$. Any derivative $\partial B_{pq}/\partial y^r$ appearing in the right member of an equation (h) is such that the inequalities $1 \leqq q \leqq n-2$, $2 \leqq r \leqq n$, and $r > q$ are satisfied; in fact, if these inequalities were not satisfied the derivative $\partial B_{pq}/\partial y^r$ would stand in the left member and not in the right member of one of the equations (h).

Let us now denote by

$$\Phi_{lm}\left(\begin{array}{l} l = 1, \ldots, B_1 - n \text{ for } m = 1 \\ \\ l = 1, \ldots, B_m \text{ for } m = 2, \ldots, n-2 \end{array}\right) \qquad \text{(i)}$$

an arbitrary function of the variables $y^{m+1}, \ldots, y^n$ analytic in the neighborhood of the values $y^{m+1} = \ldots = y^n = 0$ and let us put $B_{lm} = \Phi_{lm}$ for $y^1, \ldots, y^m = 0$. These components B_{lm} are therefore determined at $y^i = 0$ and from them, from equations (b) and (c), and from the fact that $g_{\alpha\beta} = \pm\delta^\alpha_\beta$ at $y^i = 0$, the remaining components $g_{\alpha\beta,\gamma\delta}$ at $y^i = 0$ are determined. In fact, it is easily seen by dividing (b) into three separate systems corresponding to (I), (II) and (III) that the above components B_{lm} determine the remaining independent components B after which the dependent components $g_{\alpha\beta,\gamma\delta}$ can be determined from (c). If we denote a derivative of any component B_{lm} of order p evaluated at $y^i = 0$ by $(B_{lm,\sigma_1\ldots\sigma_p})_0$ then the number of derivatives $(B_{lm,\sigma_1\ldots\sigma_p})_0$ whose values are determined as a consequence of equations (b), is *at most* equal to

$$K(n,2)\; K(n,p) - nK(n,p-1). \qquad \text{(j)}$$

Hence the number of *arbitrary* derivatives $(B_{lm,\sigma_1\ldots\sigma_p})_0$ is given by the expression

$$[B_1 - n]\, K(n-1,p) + B_2K(n-2,p) + \ldots + B_{n-2}K(2,p), \qquad \text{(k)}$$

and the remaining derivatives $(B_{lm,\sigma_1\ldots\sigma_p})_0$ can be found from (h) and the equations obtainable from (h) by successive differentiation.[3] In fact, if the number of arbitrary derivatives $(B_{lm,\sigma_1\ldots\sigma_p})_0$ were less than (k), then the number of arbitrary derivatives $(B_{lm,\sigma_1\ldots\sigma_p})_0$ in the general metric

space defined by the quadratic differential form $g_{\alpha\beta}\,dx^\alpha\,dx^\beta$ would be less than the sum of (j) and (k), i. e.,

$$B_0K(n,p) + B_1K(n-1,p) + \ldots + B_{n-2}K(2,p),$$

or $N(n, 2+p)$ which is not possible.[5] Hence the coefficients in the power series expansion about the origin $y^i = 0$ of any independent component B_{lm} have a unique determination; if this were not the case there would exist an integer p ($\geqq 2$) as the result of a condition of integrability such that the number of arbitrary derivatives $(B_{lm,\sigma_1\ldots\sigma_p})_0$ would be less than the number (k). Since any dependent component $g_{\alpha\beta,\gamma\delta}$ is given by (c) as a simple linear expression in the independent components B_{lm}, the power series expansion of any component $g_{\alpha\beta,\gamma\delta}$, namely,

$$g_{\alpha\beta,\gamma\delta} = (g_{\alpha\beta,\gamma\delta})_0 + (g_{\alpha\beta,\gamma\delta,\epsilon})_0 y^\epsilon + \ldots, \qquad (l)$$

is uniquely determined. The proof of the convergence of the series (l) will be given later as part of a systematic treatment of the existence of integrals of systems of tensor differential equations. Assuming for the present the convergence of the expansions (l) it can be shown that there exists a fundamental metric tensor with components $g_{\alpha\beta}$ (y) with respect to which (l) the functions $g_{\alpha\beta,\gamma\delta}$ defined by (1) are the components of the second extension, and (2) the variables y^i, on which the components $g_{\alpha\beta}$ depend, are the coördinates of a system of normal coördinates.[3] As it is obvious that these components $g_{\alpha\beta}$ (y) constitute a set of integrals of (b) we have the

Existence Theorem.—Let Φ_{lm} with range of indices specified by (i), *denote an arbitrary function of the variables $y^{m+1}, \ldots, y^n$ analytic in the neighborhood of the values $y^1 = \ldots = y^m = 0$. Also let $g_{\alpha\beta}$ $(= g_{\beta\alpha})$ denote the components of a fundamental metric tensor T in a system of normal coördinates y^i, each function $g_{\alpha\beta}$ (y) being analytic in the neighborhood of the values $y^i = 0$. Then there exists one, and only one, set of components $g_{\alpha\beta}$ of T which constitutes a set of integrals of the system of equations* (b) *and which is* (1) *such that $g_{\alpha\beta} = \pm\,\delta^\alpha_\beta$ at $y^i = 0$, and* (2) *such that the components B_{lm} of the resulting second extension are:*

$$B_{l1} = \Phi_{l1}\ (l = 1, \ldots, B_1 - n)$$

for $y^1 = 0$; and

$$B_{lm} = \Phi_{lm}\ (l = 1, \ldots, B_m;\ m = 2, \ldots, n-2)$$

for $y^1 = \ldots = y^m = 0$.

In particular if $n = 4$ in the above theorem we have the following

Existence theorem for Einstein's gravitational equations. Let

$$\mathfrak{A}\,(y^2,y^3,y^4);\ \mathfrak{B}\,(y^2,y^3,y^4);\ \mathfrak{C}\,(y^2,y^3,y^4);\ \mathfrak{D}\,(y^2,y^3,y^4)$$

denote arbitrary functions of the three variables y^2,y^3,y^4 and let

$$\mathfrak{P}(y^3,y^4);\ \mathfrak{Q}(y^3,y^4);\ \mathfrak{R}(y^3,y^4)$$
$$\mathfrak{S}(y^3,y^4);\ \mathfrak{T}(y^3,y^4);\ \mathfrak{U}(y^3,y^4)$$

denote arbitrary functions of the two variables y^3,y^4; *each function* $\mathfrak{A}, \ldots, \mathfrak{U}$ *is assumed to be analytic in the neighborhood of the values* $y^i = 0$ *of its arguments. Also let* $g_{\alpha\beta}$ $(= g_{\beta\alpha})$ *denote the components of a fundamental metric tensor* T *in a system of normal coördinates* y^i, *each function* $g_{\alpha\beta}(y)$ *being analytic in the neighborhood of the values* $y^i = 0$. *Then there exists one, and only one, set of components* $g_{\alpha\beta}$ *of* T *which constitutes a set of integrals of Einstein's gravitational equations* (*b*) *and which is* (1) *such that*

$$g_{11} = 1;\ g_{22} = g_{33} = g_{44} = -1;$$
$$g_{\alpha\beta} = 0, \text{ if } \alpha \neq \beta;$$

for $y^1 = \ldots = y^4 = 0$, *and* (2) *such that*

$$g_{21,34} = \mathfrak{A}(y^2,y^3,y^4);\ g_{21,44} = \mathfrak{B}(y^2,y^3,y^4);$$
$$g_{22,34} = \mathfrak{C}(y^2,y^3,y^4);\ g_{22,44} = \mathfrak{D}(y^2,y^3,y^4);$$

for $y^1 = 0$ *and*

$$g_{31,24} = \mathfrak{P}(y^3,y^4);\ g_{31,34} = \mathfrak{Q}(y^3,y^4);\ g_{31,44} = \mathfrak{R}(y^3,y^4);$$
$$g_{32,34} = \mathfrak{S}(y^3,y^4);\ g_{32,44} = \mathfrak{T}(y^3,y^4);\ g_{33,44} = \mathfrak{U}(y^3,y^4)$$

for $y^1 = y^2 = 0$. In the above existence theorem for Einstein's gravitational equations (b) we have identified y^1 with the time coördinate and y^2,y^3,y^4 with the space coördinates.[6] Any one of the variables y^i can, however, be identified with the time coördinate after which the remaining variables are to be identified with the space coördinates. There is also a certain amount of arbitrariness in the selection of the arbitrary components $g_{\alpha\beta,\gamma\delta}$, although this selection cannot be made entirely at pleasure owing to the identities (c) and (d) as well as the conditions imposed by the differential equations (b) themselves.

[1] K. Schwarzschild, "Uber das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie," *Berlin, Sitzungsberichte*, 189 (1916).

[2] H. Weyl, "Zur Gravitationstheorie," *Ann. Physik*, **54**, 117 (1917); also, *Ibid.*, **59**, 185 (1919).

[3] T. Y. Thomas, "The Existence Theorems in the Problem of the Determination of Affine and Metric Spaces by Their Differential Invariants," not yet published.

[4] T. Y. Thomas, "The Identities of Affinely Connected Manifolds," *Math. Zeit.*, **25**, 714 (1926). In this article it is shown that any identity satisfied by the components $g_{\alpha\beta,\gamma\delta}$ can be derived from the identities (*c*). The identity $g_{\alpha\beta,\gamma\delta} = g_{\gamma\delta,\alpha\beta}$, derivable from (*c*), is an important one which if kept in mind will facilitate the reading of the present note.

[5] T. Y. Thomas, loc. cit. 3 and 4; also, T. Y. Thomas and A. D. Michal, "Differential Invariants of Relative Quadratic Differential Forms," *Ann. Math.*, **28**, 631, §9 (1927).

[6] T. Y. Thomas, "The Principle of Equivalence in the Theory of Relativity," *Phil. Mag.*, **48**, 1056 (1924).

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